Appendix A: Brief Overview of Time Series Concepts

The following concepts are presented at a high level to support interpretation of the models and results in this paper, rather than as a comprehensive treatment of time series methods.

The formulations discussed in this appendix are primarily based on the structural time series and state-space modeling frameworks described in Harvey (1989), Durbin and Koopman (2012), and Scott and Varian (2014). Readers seeking a more comprehensive theoretical treatment are referred to those sources.

Drift Model

A drift model assumes that a time series evolves according to a constant average change over time.

$y_t = y_{t-1} + \delta + \varepsilon_t$

where δ represents the average drift and ε_t is random noise. Forecasts are generated by extrapolating the estimated drift forward. In this paper, the drift model serves as a simple baseline representing steady, unstructured growth without explicitly modeling structural changes.

AR(1) (First-Order Autoregressive Process)

An AR(1) process assumes that the current value depends on its immediately preceding value:

$y_t = \varphi y_{t-1} + \varepsilon_t$

where $|\varphi| < 1$ controls the degree of persistence. Values of φ close to 1 indicate strong persistence, while smaller values imply faster mean reversion. In this paper, AR(1) components capture short-term dependence and gradual adjustment in loss development patterns.

State Space Models

State space models represent observed data as arising from latent (unobserved) states that evolve over time (Harvey 1989; Durbin and Koopman 2012).

$y_t = Z_t \alpha_t + \varepsilon_t$

$\alpha_t = T_t \alpha_{t-1} + \eta_t$

where α_t represents latent components (e.g., level, trend), and ε_t, η_t are noise terms. This framework allows complex series to be decomposed into interpretable elements while accounting for uncertainty in both observed data and underlying structure.

Local Level and Trend Models

A local level model assumes the following state-space form (Harvey 1989):

$$y_t = \mu_t + \varepsilon_t$$

$$\mu_t = \mu_{t-1} + \eta_t$$

A local linear trend model extends this:

$$y_t = \mu_t + \varepsilon_t$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t$$

$$\beta_t = \beta_{t-1} + \zeta_t$$

where μ_t represents a slowly evolving baseline and β_t a time-varying slope. In reserving applications, these components capture evolving claim levels and trends over time.

Structural Breaks (Intervention Effects)

Structural breaks represent discrete changes in the level or behavior of a time series.

$$y_t = \mu_t + \gamma \cdot I(t \geq t_0) + \varepsilon_t$$

where $I(\cdot)$ is an indicator function and t_0 denotes the break point. In BSTS models, such intervention effects allow the model to capture abrupt shifts distinct from gradual trends. In BSTS frameworks, these intervention effects are commonly represented through regression-style state components or step-function covariates (Scott and Varian 2014).

Seasonality

Seasonality refers to recurring patterns at fixed intervals.

$$y_t = \mu_t + s_t + \varepsilon_t$$

where s_t follows a periodic structure (e.g., $s_t = s_{t-m}$ for period m). In reserving data, seasonal effects may reflect reporting cycles or periodic claim behavior.

Latent vs. Observed Components

In state space models, observed data y_t are expressed as functions of latent states α_t. The decomposition produced by BSTS corresponds to estimates of these latent components, typically obtained via filtering and smoothing. These components are not directly observed and should be interpreted as model-based estimates rather than definitive quantities.

Component Identifiability

Component identifiability refers to whether a unique decomposition exists for a given observed series. In practice, different combinations of components (e.g., level vs. trend) can produce similar values of y_t, particularly when the series is short or noisy. As a result, posterior distributions over components may exhibit overlap, reflecting ambiguity in attribution. This limitation is central to interpreting BSTS results in this paper.

Bayesian Inference and MCMC

BSTS models are estimated within a Bayesian framework, where parameters and latent states are assigned prior distributions and updated using observed data. The posterior distribution is typically approximated using Markov Chain Monte Carlo (MCMC) methods, which generate samples from the posterior. This allows uncertainty in both parameters and latent components to be explicitly quantified. Scott and Varian (2014) provide a practical overview of this formulation within the BSTS context.

Spike-and-Slab Priors

Spike-and-slab priors are Bayesian variable-selection priors commonly used in regression and structural time series models. The prior consists of two components:

1. a “spike” distribution concentrated near zero, representing exclusion or negligible effect, and
2. a broader “slab” distribution representing meaningful inclusion.

In BSTS models, spike-and-slab priors are often applied to candidate structural break variables or regression components, allowing the model to automatically select which effects are supported by the data while shrinking insignificant effects toward zero. This provides a mechanism for controlling model complexity and reducing overfitting in short or noisy time series (Scott and Varian 2014).

Appendix B:

Real World Data Used for Experiments 2 and 3

Note: The loss triangles below are presented on a cumulative loss and LAE basis, while the time series used in the corresponding experiments are derived from incremental development amounts.

Experiment 2:

Line of Business: Workers' Compensation (Indemnity)

Source: Anonymous self-insured entity's accident year triangle of incurred losses & LAE with small modifications

	Development Age (Months)					
Accident Year	12	24	36	48	60	72
2008	564,817	1,560,836	2,236,308	2,816,670	3,204,864	2,968,706
2009	355,440	677,676	921,799	985,268	1,140,819	1,241,176
2010	493,501	1,482,743	2,400,463	3,014,132	3,908,070	4,635,518
2011	482,129	880,013	1,186,952	1,178,112	1,199,865	1,233,799
2012	382,581	942,484	1,137,951	1,489,113	1,471,124	1,550,463
2013	256,490	1,553,317	2,256,441	2,668,147	2,561,186	2,383,514
2014	586,380	2,577,107	3,302,573	3,910,418	3,891,788	3,934,464
2015	1,124,590	1,590,446	2,148,554	2,500,521	3,355,689	3,210,763
2016	737,267	1,264,157	1,710,177	2,431,845	1,654,954	1,692,263
2017	433,379	1,041,845	1,825,528	2,190,372	2,169,275	1,803,664
2018	277,628	425,379	508,215	533,441	566,311	789,067
2019	444,939	905,577	1,342,738	1,331,976	1,345,196	1,356,731
2020	537,350	1,792,666	3,660,104	4,435,841	5,049,789	5,019,132
2021	1,138,461	3,348,094	6,009,134	7,956,599	8,442,207	
2022	2,441,641	3,961,937	6,275,670	6,736,183		
2023	690,197	2,014,040	3,249,360			
2024	511,202	1,175,922				
2025	543,861					

Experiment 3:

Line of Business: Private Passenger Auto (PPA) Bodily Injury (BI) Liability

Source: Anonymous insurer's accident quarter triangle of incurred losses & LAE with small modifications

	Development Age (Months)					
Accident Quarter	3	6	9	12	15	18
Q2-2016	445,812	787,921	852,743	938,571	957,112	955,731
Q3-2016	553,612	803,004	925,882	968,745	975,540	1,223,815
Q4-2016	390,624	669,102	676,115	771,982	788,921	865,744
Q1-2017	315,742	578,301	658,921	711,488	787,201	798,512
Q2-2017	446,087	569,321	925,774	940,812	842,210	1,142,544
Q3-2017	270,934	544,712	573,681	641,842	652,231	689,015
Q4-2017	290,811	553,488	734,012	719,004	871,554	962,881
Q1-2018	468,122	596,511	698,992	734,502	863,221	876,884
Q2-2018	266,087	399,611	581,332	675,512	922,104	898,755
Q3-2018	365,541	588,102	717,889	981,442	1,219,641	1,315,290
Q4-2018	375,102	631,821	931,244	986,552	1,009,122	1,054,877
Q1-2019	444,812	576,421	791,512	802,744	874,301	900,522
Q2-2019	205,401	462,771	635,289	731,488	741,022	758,244
Q3-2019	516,241	855,821	1,039,102	1,125,901	1,325,610	1,352,433
Q4-2019	276,511	484,322	574,022	819,911	938,201	1,019,442
Q1-2020	414,822	595,901	627,821	634,211	715,188	735,844
Q2-2020	75,612	200,441	222,844	275,011	286,221	297,244
Q3-2020	245,512	372,211	432,944	441,521	509,211	531,502
Q4-2020	348,621	602,901	647,301	723,211	836,221	
Q1-2021	498,311	793,102	1,189,211	1,370,022		
Q2-2021	281,744	428,511	613,712			
Q3-2021	248,322	679,521				
Q4-2021	346,211					

Appendix C: Full Experimental Results

Experiment 1:

Assumptions:

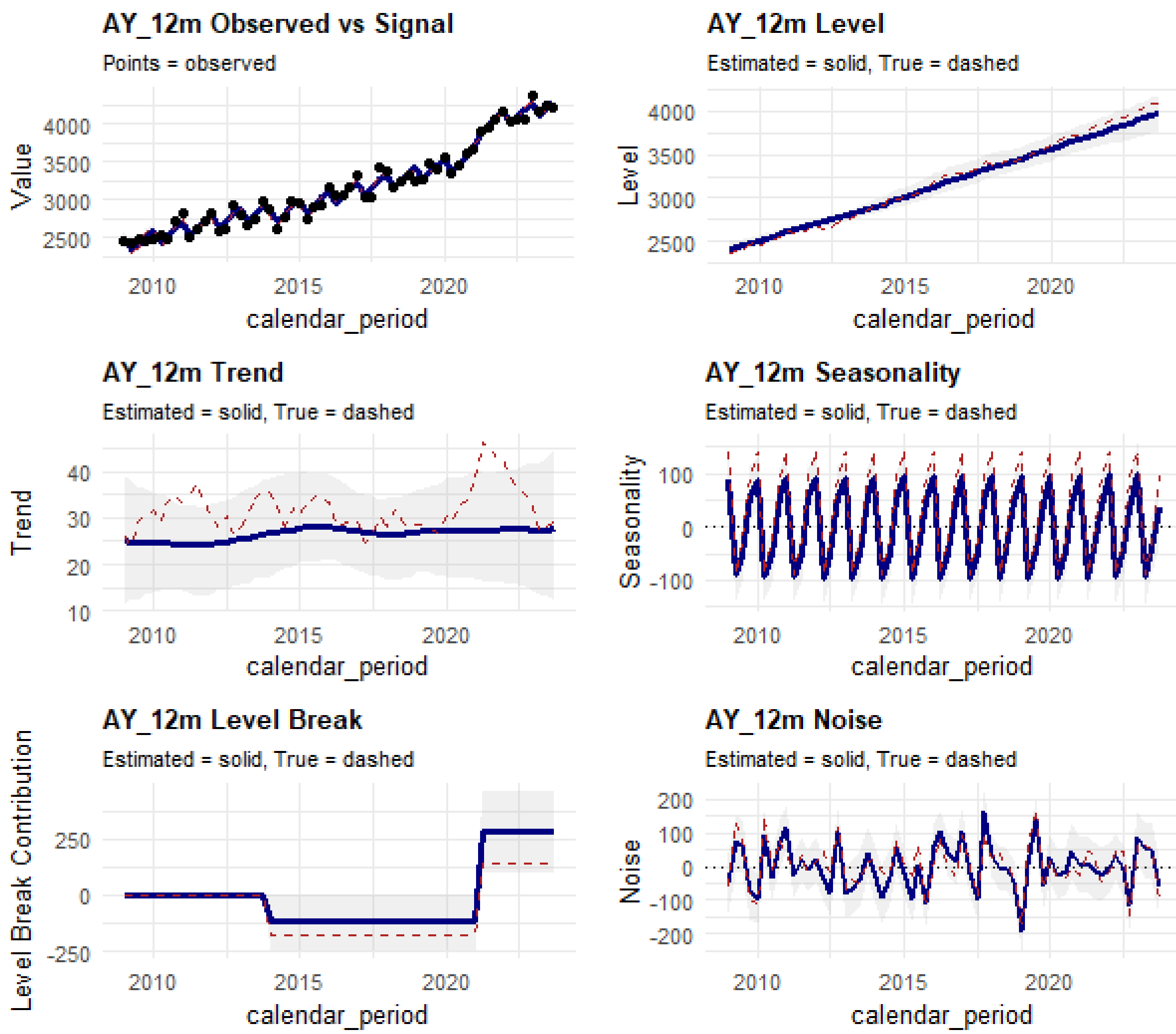
- **Model:**
  Bayesian structural time series (BSTS) with
  - Local linear trend
  - Seasonal component (quarterly, $m = 4$)
- **State Components:**
  - Level and trend evolve stochastically (local linear trend)
  - Seasonality modeled as a stochastic quarterly process
- **Structural Breaks:**
  - Two step-function regressors at ~35% and ~70% of the sample
  - Inclusion governed by spike-and-slab prior
  - Break magnitudes estimated (not fixed)
- **Observation Noise:**
  - $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$, with σ^2estimated
- **Priors:**
  - Spike-and-slab prior on break regressors (default BSTS specification)
- **Estimation:**
  - MCMC with 2,500 iterations
  - 20% burn-in (500 iterations)
- **Data Construction:**
  - Input series (CY or AY) derived deterministically from the loss triangle
  - No explicit stochastic model imposed at the triangle level

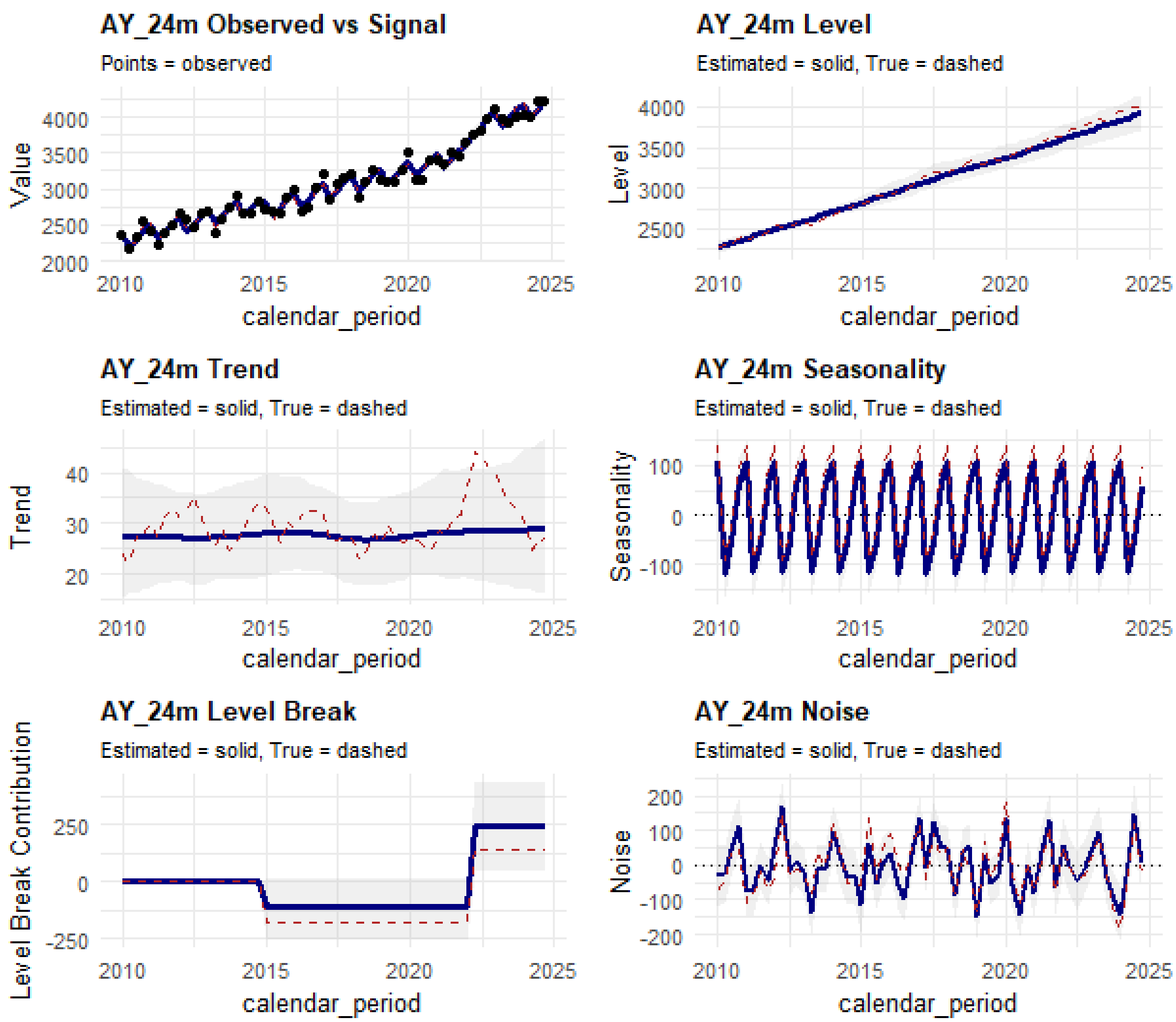
Component Estimation Accuracy Summary:

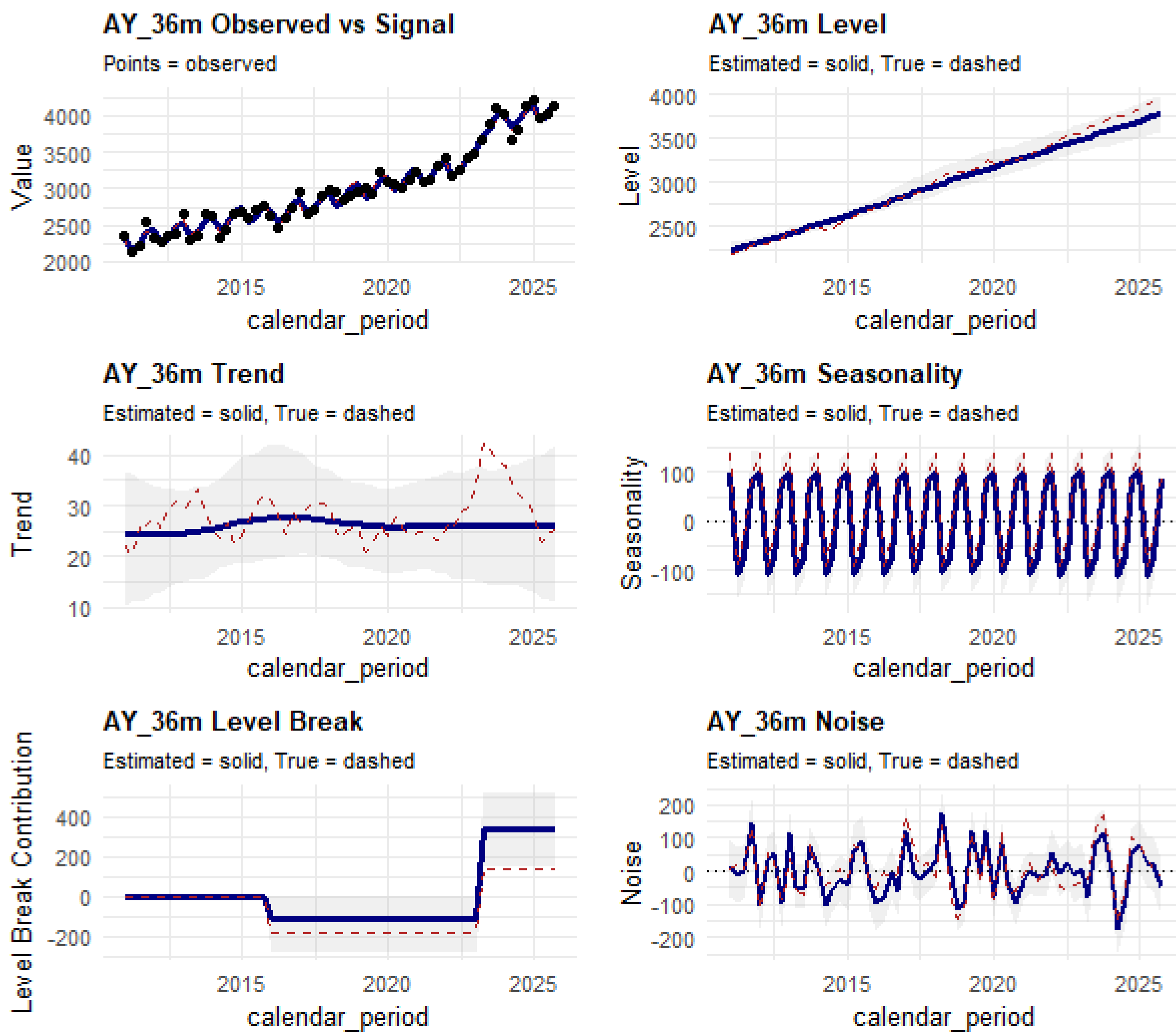
Series	Component	MAE	RMSE	MAPE
AY_12m	Level	52.639	63.688	1.601
AY_12m	Trend	5.378	6.948	15.514
AY_12m	Seasonality	28.945	34.419	36.578
AY_36m	LevelBreak	66.151	93.656	42.357
AY_36m	Signal	29.379	35.149	0.996
AY_36m	Noise	29.379	35.149	109.019
AY_72m	Level	36.073	43.891	1.233
AY_72m	Trend	6.547	7.201	33.277
AY_72m	Seasonality	28.747	29.844	48.479

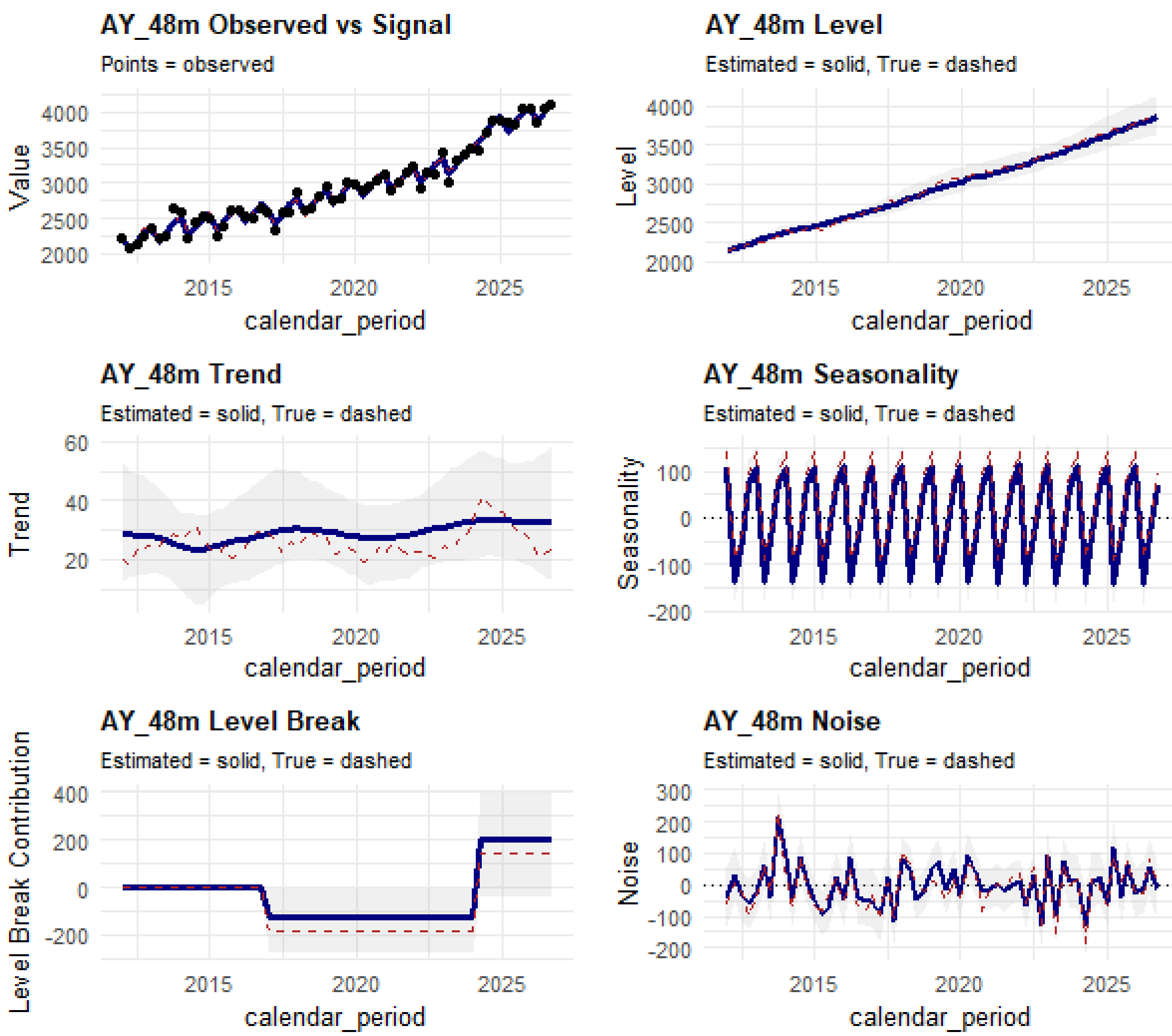
Accident Quarter	Development Age (Months)					
	3	6	9	12	15	18
Q2-2016	445,812	787,921	852,743	938,571	957,112	955,731
Q3-2016	553,612	803,004	925,882	968,745	975,540	1,223,815
Q4-2016	390,624	669,102	676,115	771,982	788,921	865,744
Q1-2017	315,742	578,301	658,921	711,488	787,201	798,512
Q2-2017	446,087	569,321	925,774	940,812	842,210	1,142,544
Q3-2017	270,934	544,712	573,681	641,842	652,231	689,015
Q4-2017	290,811	553,488	734,012	719,004	871,554	962,881
Q1-2018	468,122	596,511	698,992	734,502	863,221	876,884
Q2-2018	266,087	399,611	581,332	675,512	922,104	898,755
Q3-2018	365,541	588,102	717,889	981,442	1,219,641	1,315,290
Q4-2018	375,102	631,821	931,244	986,552	1,009,122	1,054,877
Q1-2019	444,812	576,421	791,512	802,744	874,301	900,522
Q2-2019	205,401	462,771	635,289	731,488	741,022	758,244
Q3-2019	516,241	855,821	1,039,102	1,125,901	1,325,610	1,352,433
Q4-2019	276,511	484,322	574,022	819,911	938,201	1,019,442
Q1-2020	414,822	595,901	627,821	634,211	715,188	735,844
Q2-2020	75,612	200,441	222,844	275,011	286,221	297,244
Q3-2020	245,512	372,211	432,944	441,521	509,211	531,502
Q4-2020	348,621	602,901	647,301	723,211	836,221	
Q1-2021	498,311	793,102	1,189,211	1,370,022		
Q2-2021	281,744	428,511	613,712			
Q3-2021	248,322	679,521				
Q4-2021	346,211					

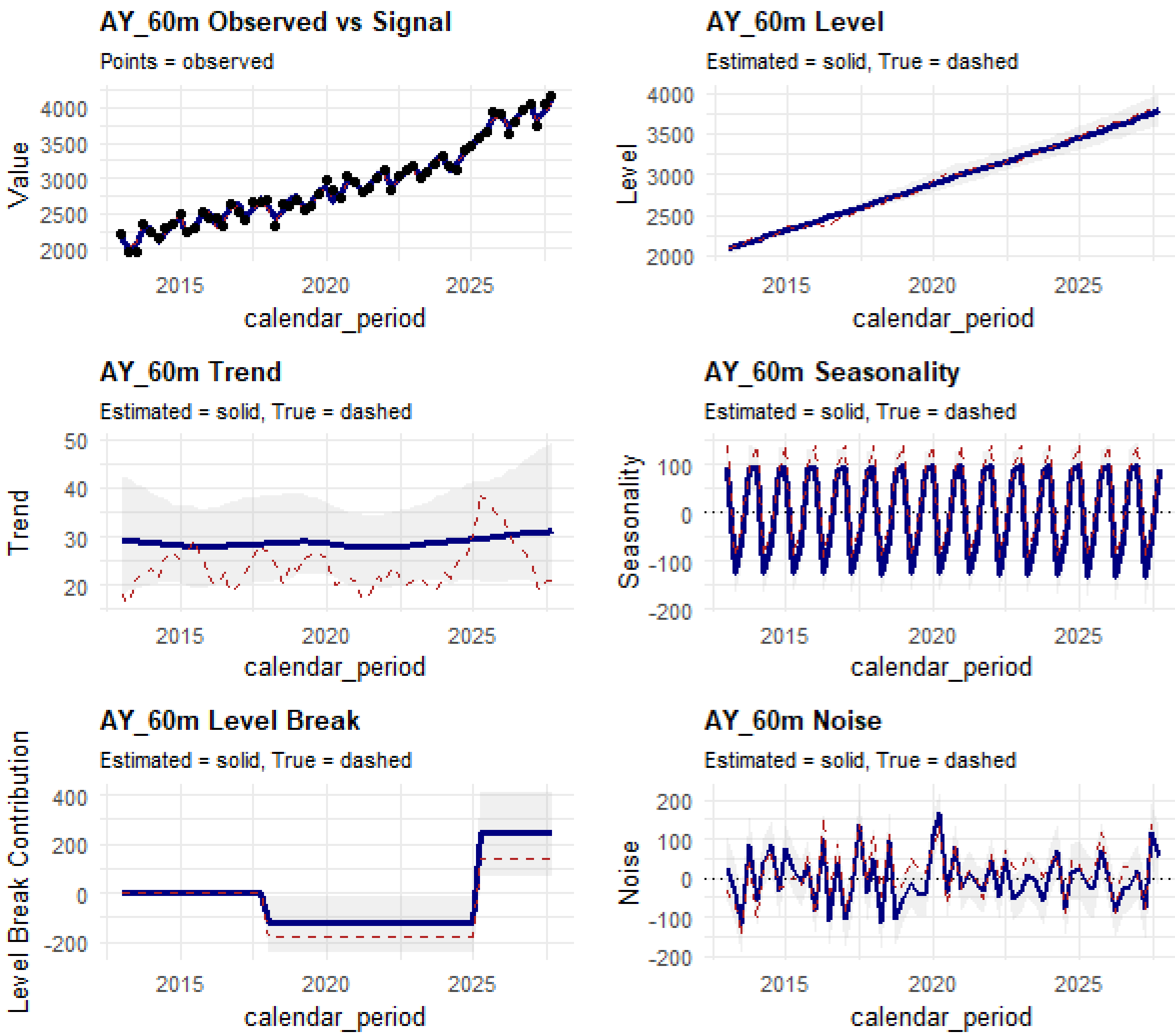
Decomposition Plots:

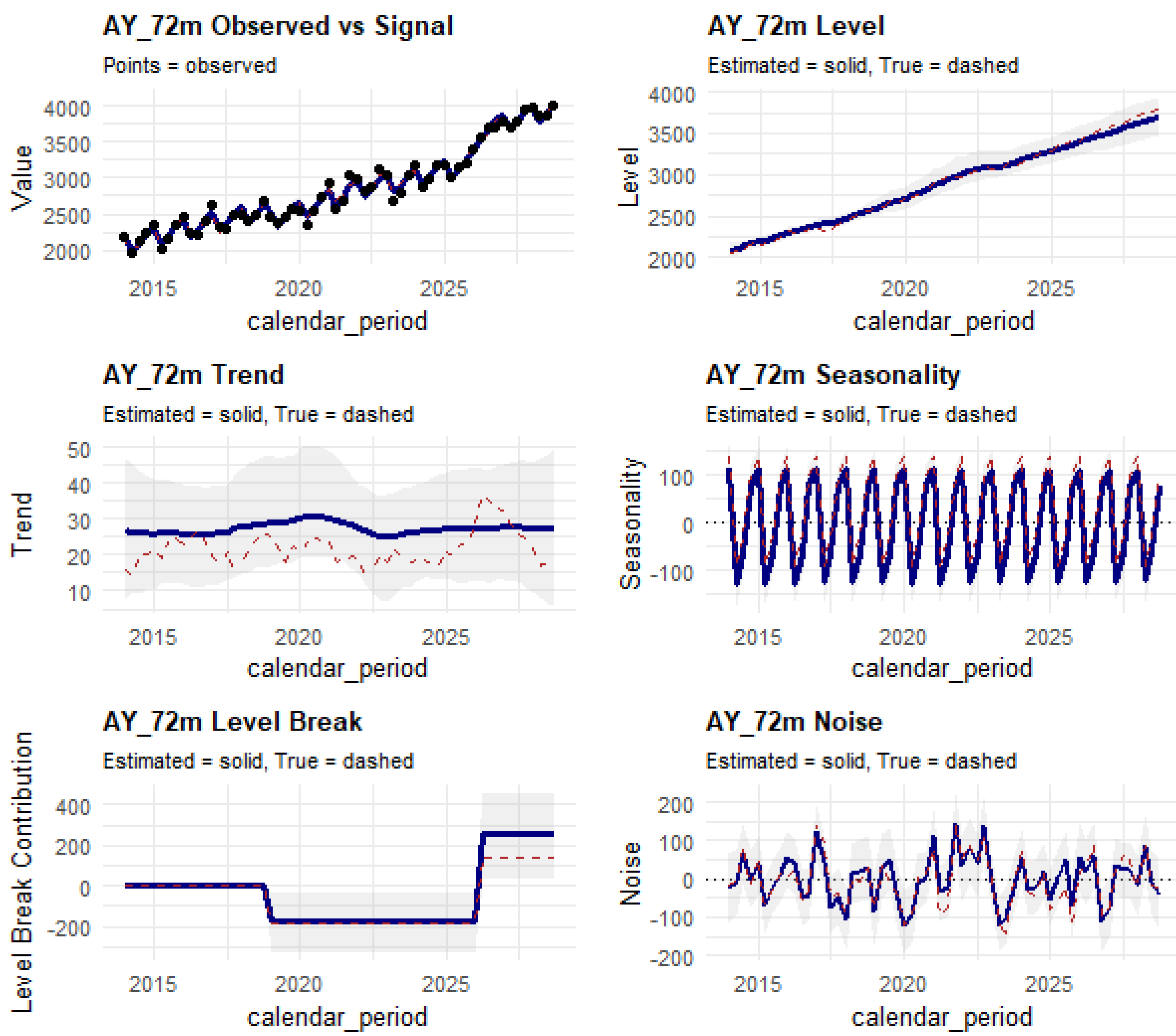


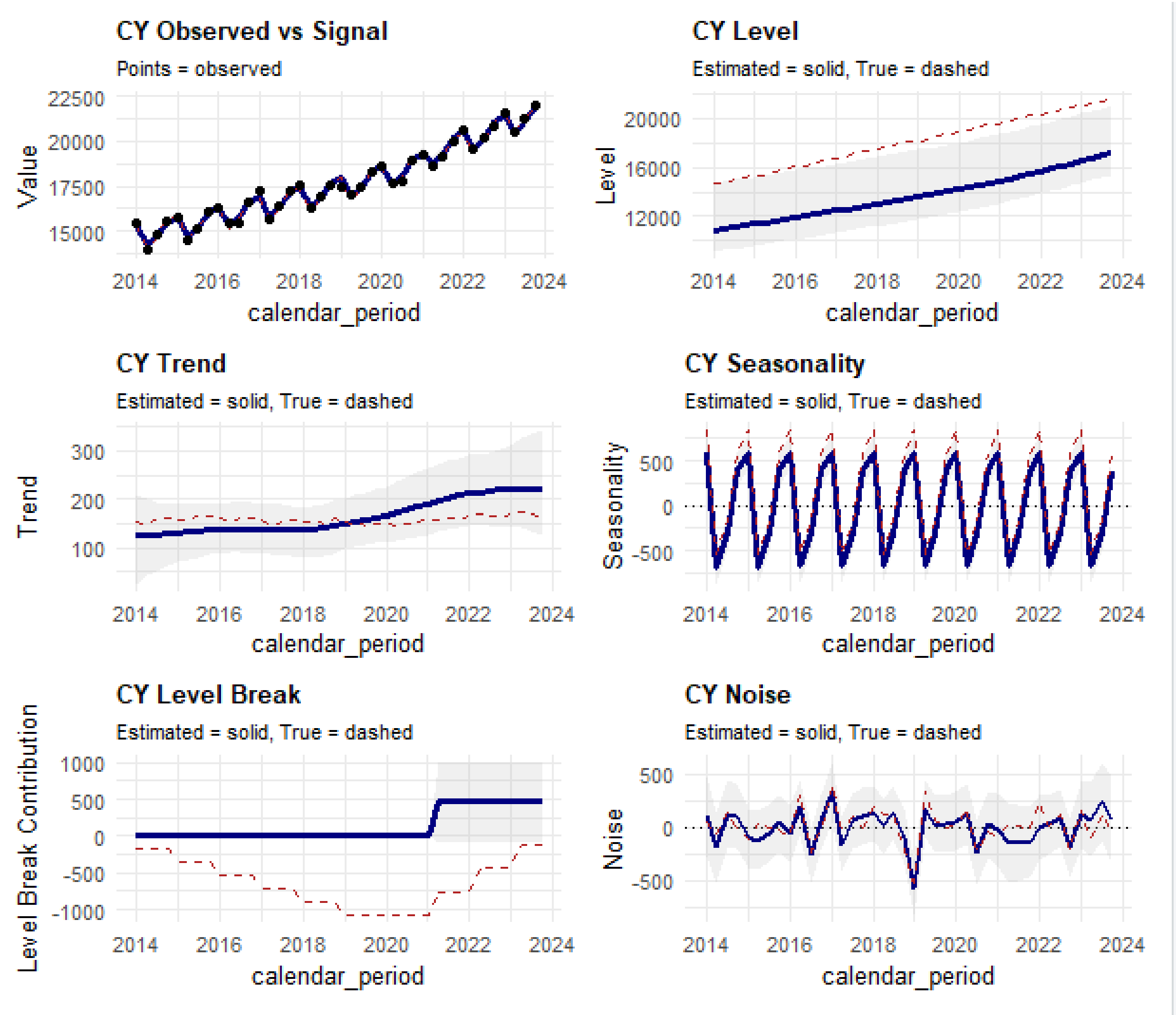

AY_24m Observed vs Signal
Points = observed
Value
calendar_period
AY_24m Level
Estimated = solid, True = dashed
Level
calendar_period
AY_24m Trend
Estimated = solid, True = dashed
Trend
calendar_period
AY_24m Seasonality
Estimated = solid, True = dashed
Seasonality
calendar_period
AY_24m Level Break
Estimated = solid, True = dashed
Level Break Contribution
calendar_period
AY_24m Noise
Estimated = solid, True = dashed
Noise
calendar_period


AY_36m Observed vs Signal
Points = observed
Value
4000
3500
3000
2500
2000
2015
2020
2025
calendar_period
AY_36m Level
Estimated = solid, True = dashed
Level
4000
3500
3000
2500
calendar_period
AY_36m Trend
Estimated = solid, True = dashed
Trend
40
30
20
10
calendar_period
AY_36m Seasonality
Estimated = solid, True = dashed
Seasonality
100
0
-100
calendar_period
AY_36m Level Break
Estimated = solid, True = dashed
Level Break Contribution
400
200
0
-200
calendar_period
AY_36m Noise
Estimated = solid, True = dashed
Noise
200
100
0
-100
-200
calendar_period


AY_48m Observed vs Signal
Points = observed
Value
4000
3500
3000
2500
2000
2015
2020
2025
calendar_period
AY_48m Level
Estimated = solid, True = dashed
Level
4000
3500
3000
2500
2000
2015
2020
2025
calendar_period
AY_48m Trend
Estimated = solid, True = dashed
Trend
60
40
20
2015
2020
2025
calendar_period
AY_48m Seasonality
Estimated = solid, True = dashed
Seasonality
100
0
-100
-200
2015
2020
2025
calendar_period
AY_48m Level Break
Estimated = solid, True = dashed
Level Break Contribution
400
200
0
-200
2015
2020
2025
calendar_period
AY_48m Noise
Estimated = solid, True = dashed
Noise
300
200
100
0
-100
-200
2015
2020
2025
calendar_period


AY_60m Observed vs Signal
Points = observed
Value
4000
3500
3000
2500
2000
2015
2020
2025
calendar_period
AY_60m Level
Estimated = solid, True = dashed
Level
4000
3500
3000
2500
2000
2015
2020
2025
calendar_period
AY_60m Trend
Estimated = solid, True = dashed
Trend
50
40
30
20
2015
2020
2025
calendar_period
AY_60m Seasonality
Estimated = solid, True = dashed
Seasonality
100
0
-100
-200
2015
2020
2025
calendar_period
AY_60m Level Break
Estimated = solid, True = dashed
Level Break Contribution
400
200
0
-200
2015
2020
2025
calendar_period
AY_60m Noise
Estimated = solid, True = dashed
Noise
200
100
0
-100
-200
2015
2020
2025
calendar_period


AY_72m Observed vs Signal
Points = observed
Value
calendar_period
AY_72m Level
Estimated = solid, True = dashed
Level
calendar_period
AY_72m Trend
Estimated = solid, True = dashed
Trend
calendar_period
AY_72m Seasonality
Estimated = solid, True = dashed
Seasonality
calendar_period
AY_72m Level Break
Estimated = solid, True = dashed
Level Break Contribution
calendar_period
AY_72m Noise
Estimated = solid, True = dashed
Noise
calendar_period


CY Observed vs Signal
Points = observed
Value
calendar_period
CY Level
Estimated = solid, True = dashed
Level
calendar_period
CY Trend
Estimated = solid, True = dashed
Trend
calendar_period
CY Seasonality
Estimated = solid, True = dashed
Seasonality
calendar_period
CY Level Break
Estimated = solid, True = dashed
Level Break Contribution
calendar_period
CY Noise
Estimated = solid, True = dashed
Noise
calendar_period

Experiment 2:

Assumptions:

Data/Modeling Assumptions

- **Data Transformation:**
  - Incremental losses modeled on log scale:

$$y_t = \log(\text{incremental loss}_t + 1)$$

- **Series Construction:**
  - AY series: fixed development age slices over accident year
  - CY series: diagonal aggregation of incremental triangle
- **Train / Test Split:**
  - Final 3 observations held out for forecast evaluation
  - All models fit on training data only

Forecasting Models:

- **Naive:**
  - Forecast = last observed value
- **Drift (Linear Naive):**
  - Forecast extends last observed slope
- **OLS Trend:**
  - Linear regression: $y_t = \beta_0 + \beta_1 t + \varepsilon_t$
- **BSTS – Local Level + Breaks:**
  - Local level state model
  - Step-function break regressors
  - Break inclusion learned via spike-and-slab prior
- **BSTS – Local Linear Trend + Breaks:**
  - Local linear trend state model
  - Same break structure as above
- **BSTS – Local Level + Breaks + AR(1):**
  - Local level BSTS model
  - AR(1) adjustment applied to residuals in forecast stage

Structural Break Specification (BSTS models only):

- **Candidate breakpoints:**
  - Step functions at indices spaced every 4 periods
  - Restricted to interior of training sample
- **Break inclusion:**
  - Determined via spike-and-slab prior
  - Number of active breaks inferred from posterior inclusion probabilities

---

Note about observations:

- All models operate on log-transformed scale
- Forecasts converted back via:

$$\hat{y} = \exp(\hat{y}_{\log}) - 1$$

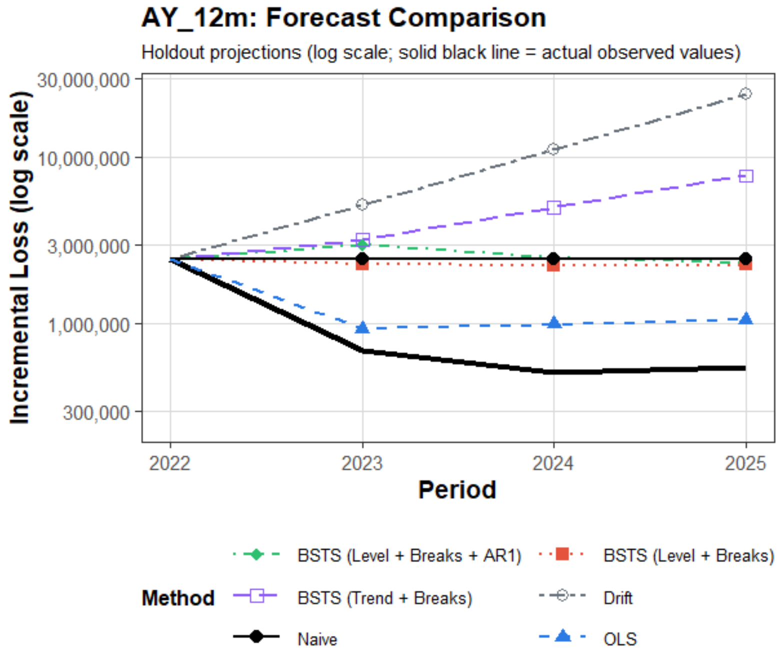
Estimation:

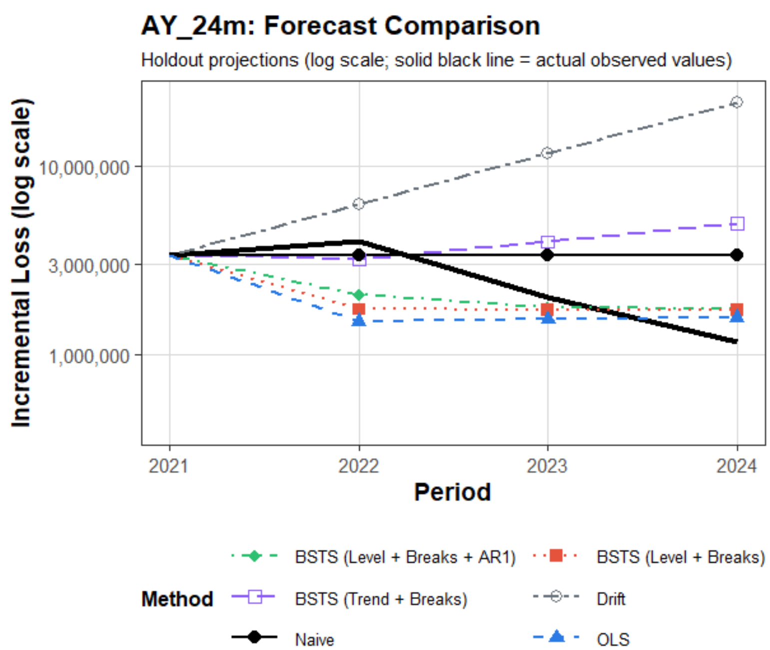
- BSTS models estimated via MCMC:
  - Iterations: **5,000**
  - Burn-in: **1,000**

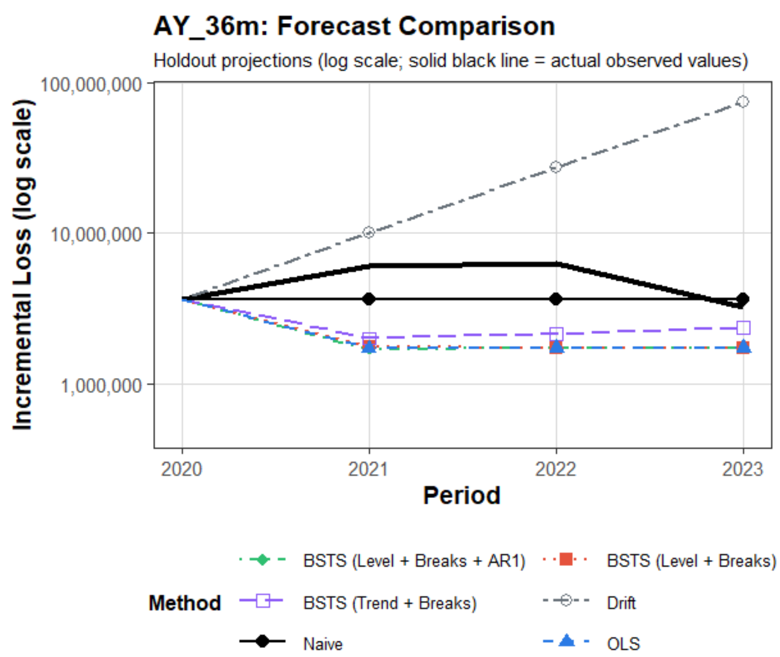
Component Estimation Accuracy Summary:

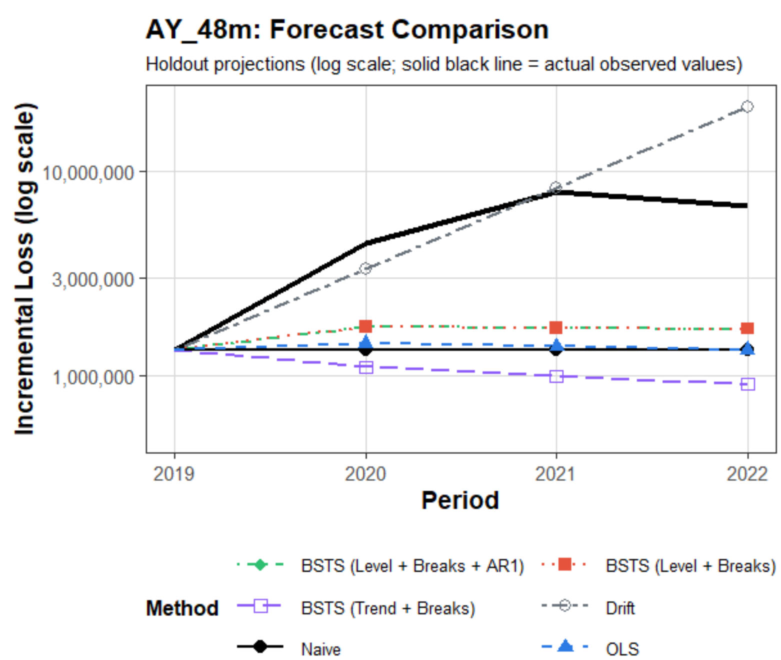
	Series	Method	Holdout Observations	MAE	MSE	**MAPE**
1	AY_12m	OLS_Trend	3	412,969	1.9.E+11	**74.7%**
2	AY_12m	BSTS_LevelBreaks	3	1,679,375	2.8.E+12	**295.2%**
3	AY_12m	Naive	3	1,859,888	3.5.E+12	**326.8%**
4	AY_12m	BSTS_LevelBreaks_AR1	3	2,017,415	4.1.E+12	**348.8%**
5	AY_12m	BSTS_TrendBreaks	3	4,746,822	2.6.E+13	**857.8%**
6	AY_12m	Drift	3	12,936,156	2.3.E+14	**2361.5%**
7	AY_24m	BSTS_LevelBreaks_AR1	3	890,731	1.3.E+12	**35.5%**
8	AY_24m	BSTS_LevelBreaks	3	1,020,967	1.8.E+12	**39.0%**
9	AY_24m	OLS_Trend	3	1,112,891	2.2.E+12	**39.8%**
10	AY_24m	Naive	3	1,373,356	2.3.E+12	**88.8%**
11	AY_24m	BSTS_TrendBreaks	3	2,168,119	6.2.E+12	**145.9%**
12	AY_24m	Drift	3	10,863,910	1.7.E+14	**764.2%**
13	AY_36m	Naive	3	1,791,780	4.2.E+12	**31.1%**
14	AY_36m	BSTS_TrendBreaks	3	3,004,761	1.1.E+13	**53.2%**
15	AY_36m	BSTS_LevelBreaks	3	3,434,410	1.4.E+13	**63.2%**
16	AY_36m	BSTS_LevelBreaks_AR1	3	3,449,020	1.4.E+13	**63.5%**
17	AY_36m	OLS_Trend	3	3,449,116	1.4.E+13	**63.5%**
18	AY_36m	Drift	3	31,923,057	1.8.E+15	**860.3%**
19	AY_48m	BSTS_LevelBreaks	3	4,658,321	2.4.E+13	**71.4%**
20	AY_48m	BSTS_LevelBreaks_AR1	3	4,659,095	2.4.E+13	**71.4%**
21	AY_48m	OLS_Trend	3	4,980,523	2.7.E+13	**76.6%**
22	AY_48m	Naive	3	5,044,232	2.8.E+13	**77.8%**
23	AY_48m	Drift	3	5,152,579	6.6.E+13	**79.1%**
24	AY_48m	BSTS_TrendBreaks	3	5,370,676	3.1.E+13	**83.0%**
25	AY_60m	BSTS_LevelBreaks	3	3,458,534	1.8.E+13	**59.5%**
26	AY_60m	OLS_Trend	3	3,637,895	2.1.E+13	**55.0%**
27	AY_60m	BSTS_LevelBreaks_AR1	3	3,678,956	1.9.E+13	**74.5%**
28	AY_60m	BSTS_TrendBreaks	3	4,277,127	2.7.E+13	**73.0%**
29	AY_60m	Naive	3	4,379,420	2.8.E+13	**80.0%**
30	AY_60m	Drift	3	4,880,227	3.3.E+13	**96.0%**
31	AY_72m	Drift	3	1,553,617	3.3.E+12	**83.7%**
32	AY_72m	Naive	3	1,558,999	3.9.E+12	**75.2%**
33	AY_72m	BSTS_TrendBreaks	3	1,617,418	4.0.E+12	**80.6%**
34	AY_72m	OLS_Trend	3	1,695,285	3.6.E+12	**99.6%**
35	AY_72m	BSTS_LevelBreaks	3	1,722,056	3.6.E+12	**103.3%**
36	AY_72m	BSTS_LevelBreaks_AR1	3	1,760,357	3.7.E+12	**109.5%**
37	CY	Naive	3	8,702,233	8.7.E+13	**38.2%**
38	CY	BSTS_TrendBreaks	3	9,903,702	1.1.E+14	**44.1%**
39	CY	Drift	3	10,196,012	1.6.E+14	**43.0%**
40	CY	OLS_Trend	3	10,677,931	1.3.E+14	**47.5%**
41	CY	BSTS_LevelBreaks_AR1	3	10,907,017	1.3.E+14	**48.5%**
42	CY	BSTS_LevelBreaks	3	10,935,779	1.3.E+14	**48.7%**

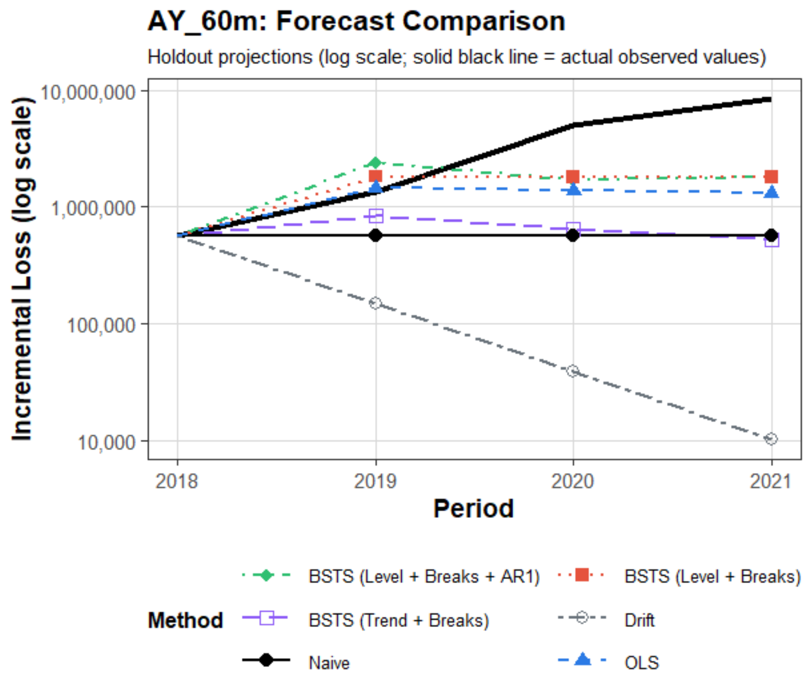
Forecast Comparison Plots:



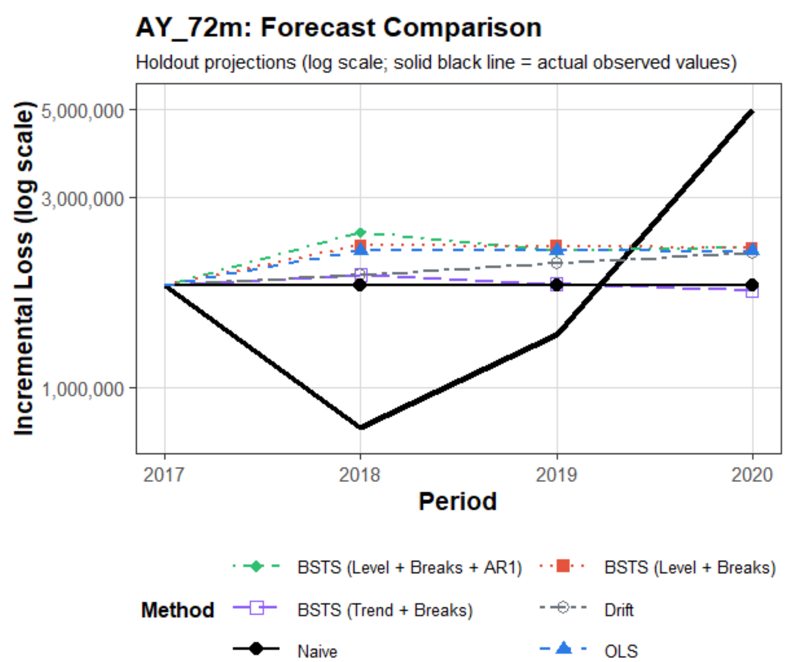



AY_36m: Forecast Comparison
Holdout projections (log scale; solid black line = actual observed values)
100,000,000
10,000,000
1,000,000
Incremental Loss (log scale)
2020
2021
2022
2023
Period
Method
BSTS (Level + Breaks + AR1)
BSTS (Level + Breaks)
BSTS (Trend + Breaks)
Drift
Naive
OLS

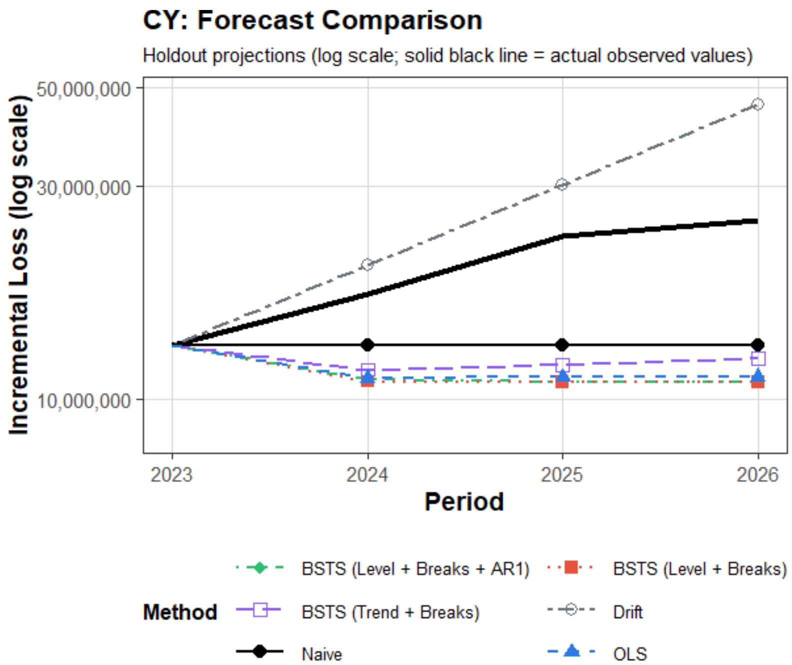

AY_48m: Forecast Comparison
Holdout projections (log scale; solid black line = actual observed values)
10,000,000
3,000,000
1,000,000
Incremental Loss (log scale)
2019
2020
2021
2022
Period
Method
BSTS (Level + Breaks + AR1)
BSTS (Level + Breaks)
BSTS (Trend + Breaks)
Drift
Naive
OLS



AY_60m: Forecast Comparison
Holdout projections (log scale; solid black line = actual observed values)
Incremental Loss (log scale)
10,000,000
1,000,000
100,000
10,000
2018
2019
2020
2021
Period
Method
BSTS (Level + Breaks + AR1)
BSTS (Level + Breaks)
BSTS (Trend + Breaks)
Drift
Naive
OLS



AY_72m: Forecast Comparison
Holdout projections (log scale; solid black line = actual observed values)
Incremental Loss (log scale)
5,000,000
3,000,000
1,000,000
2017
2018
2019
2020
Period
Method
BSTS (Level + Breaks + AR1)
BSTS (Level + Breaks)
BSTS (Trend + Breaks)
Drift
Naive
OLS


CY: Forecast Comparison
Holdout projections (log scale; solid black line = actual observed values)
Incremental Loss (log scale)
50,000,000
30,000,000
10,000,000
2023
2024
2025
2026
Period
Method
BSTS (Level + Breaks + AR1)
BSTS (Level + Breaks)
BSTS (Trend + Breaks)
Drift
Naive
OLS

Experiment 3:

Assumptions:

Data / Modeling Assumptions:

Series Construction:

- One selected series is analyzed at a time:
  - AY series: fixed development age slice from the **cumulative** triangle
  - CY series: diagonal aggregation of the **incremental** triangle derived from the cumulative triangle
- Supported series forms:
  - AY_3m, AY_6m, AY_9m, …
  - CY
- Series labels are constructed as quarterly periods beginning from a user-specified start year and quarter
- Minimum required series length: **6 observations**
- Quarterly frequency assumed: $m = 4$

Data Scale:

- Series are modeled on the **original scale**
- No log transformation is applied (USE_LOG_SCALE = FALSE)

Baseline BSTS Decomposition

- Baseline model fitted to the selected series before any perturbation
- BSTS specification includes:
  - **Local linear trend**
  - **Quarterly seasonal component**
  - **Step-function structural break regressors**
- Candidate break regressors:
  - Constructed as step functions over the time index
  - Breakpoints placed from approximately **25% to 75%** of the sample
  - Spaced every **3 periods**
- Break inclusion:
  - Determined using a **spike-and-slab prior**
  - Expected model size = **1**

State / Prior Assumptions

- **Level component:** stochastic local level within a local linear trend model
- **Trend component:** stochastic slope component
- **Seasonality:** stochastic quarterly seasonal component
- **Break effects:** additive regression effects from candidate step dummies
- **Noise:** residual component defined as observed minus fitted
- State priors are data-scaled:
  - Level sigma prior: based on series standard deviation
  - Slope sigma prior: based on series standard deviation
  - Seasonal sigma prior: based on series standard deviation

Perturbation Design
Single-component perturbations are applied to the baseline decomposition in four separate stages:

- **Stage 1 – Level perturbation:**
  - Level increased by **10%** beginning at the intervention point
- **Stage 2 – Trend perturbation:**
  - Trend perturbed by a ramp shock with size **20,000** beginning at the intervention point
  - Synthetic observed series altered using the **cumulative effect** of the trend perturbation
- **Stage 3 – Seasonality perturbation:**
  - Seasonal component amplified by a factor of **5** beginning at the intervention point
- **Stage 4 – Noise perturbation:**
  - Noise shock generated using Gaussian noise with standard deviation multiplied by **2.0** beginning at the intervention point

Intervention timing:

- For Experiment 4, intervention dates were specified explicitly rather than selected automatically:
- The level perturbation was introduced beginning in 2022Q1, the trend perturbation beginning in 2021Q3, the seasonality perturbation beginning in 2020Q1, and the noise perturbation beginning in 2021Q3. These dates were selected to ensure adequate pre-intervention history and sufficient post-intervention observations for decomposition and attribution analysis.

Refit / Attribution Framework

- After each perturbation, a new synthetic observed series is constructed
- The same BSTS decomposition is then refit to the perturbed series
- Attribution is evaluated by comparing:
  - baseline decomposition
  - ground-truth perturbed decomposition
  - refit decomposition on the perturbed series
- Component attribution is summarized using:
  - **Recovery percentage**
  - **Signal-share percentage**
  - **Allocation percentage**
- Final matrix-style summary reports:
  - rows = stressed component
  - columns = decomposed component
  - diagonal = recovery error
  - off-diagonal = cross-component leakage

Estimation

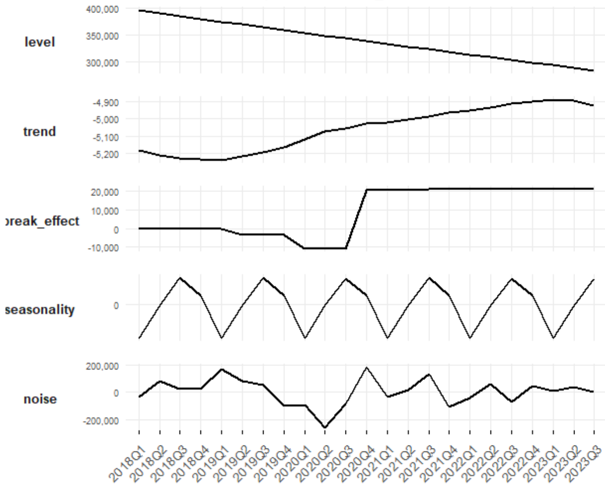
- BSTS estimated via MCMC:
  - Iterations: **2,500**
  - Burn-in: **20%** (**500** iterations)
  - Seed: **123**

Perturbation Decomposition Results by Series:

AY (3 months):

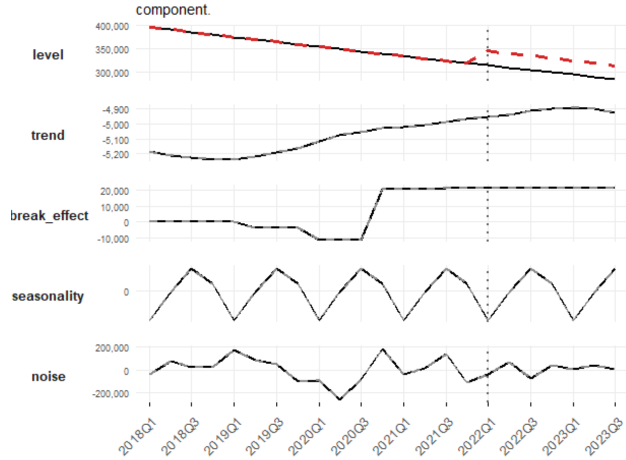
Figure 3.1 - Baseline BSTS Decomposition of AY_3m

Baseline BSTS decomposition of the AY_3m series.



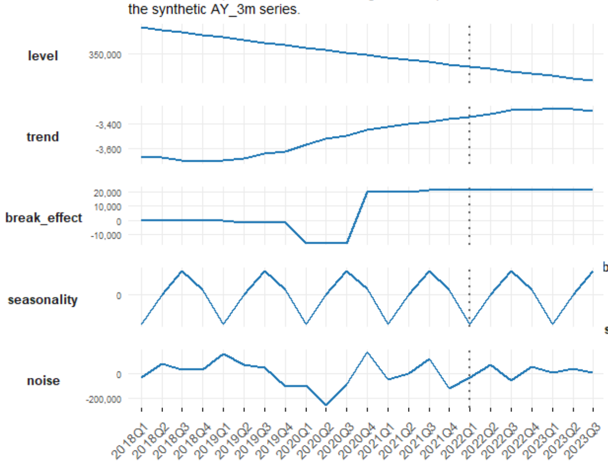
Ground-Truth Perturbation Decomposition (AY_3m | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for AY_3m. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



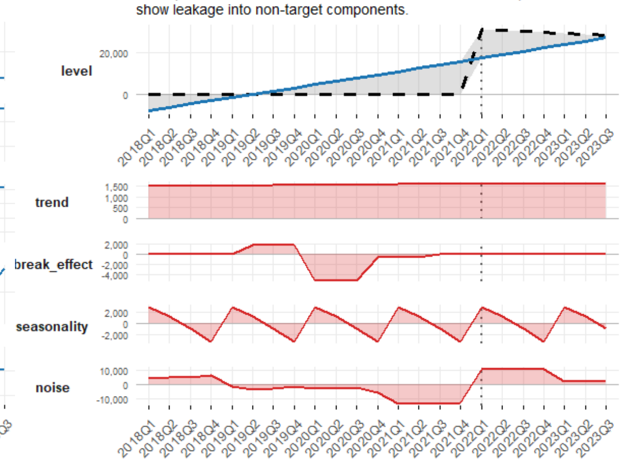
BSTS Decomposition of Perturbed Synthetic AY_3m Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic AY_3m series.



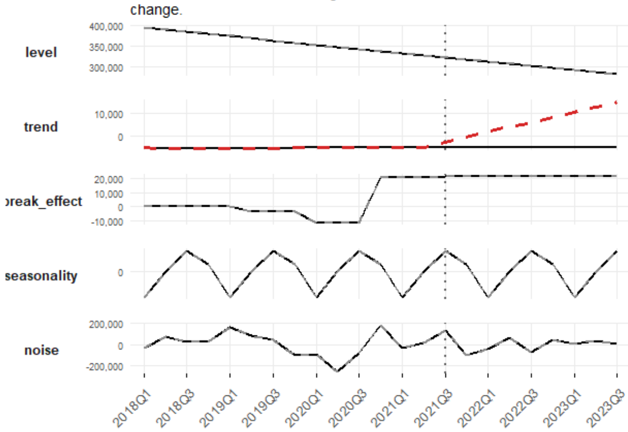
Difference Decomposition: Estimated Attribution versus True Level Perturbation (AY_3m | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



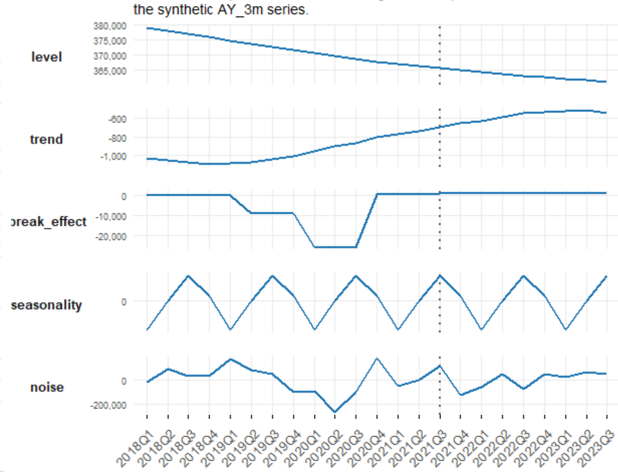
Ground-Truth Perturbation Decomposition (AY_3m | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for AY_3m. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



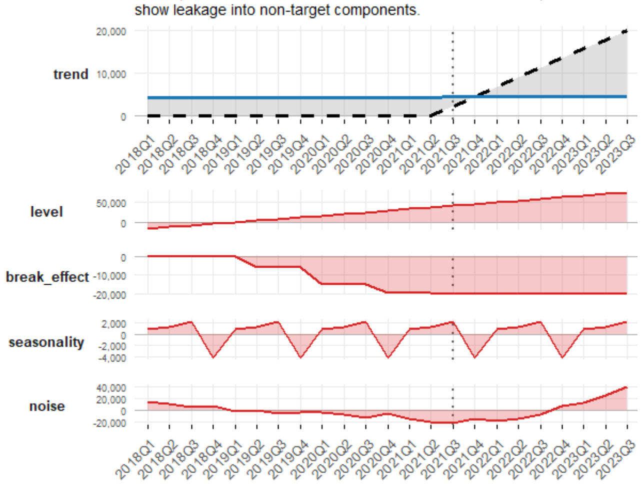
BSTS Decomposition of Perturbed Synthetic AY_3m Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic AY_3m series.



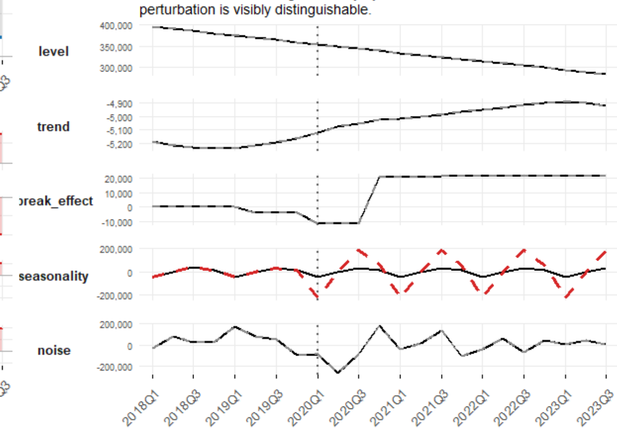
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (AY_3m | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



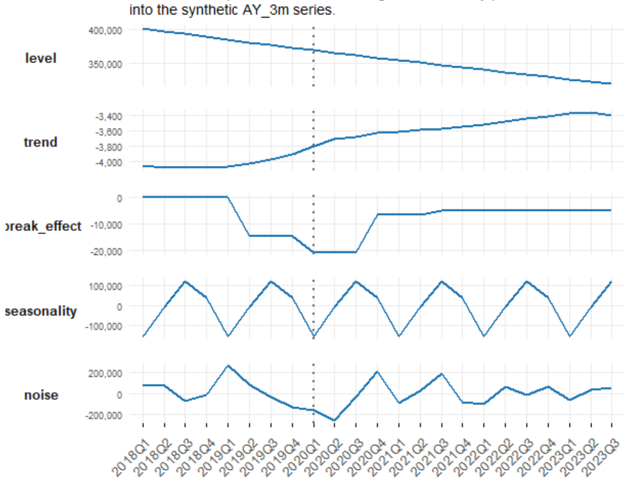
Ground-Truth Perturbation Decomposition (AY_3m | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for AY_3m. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



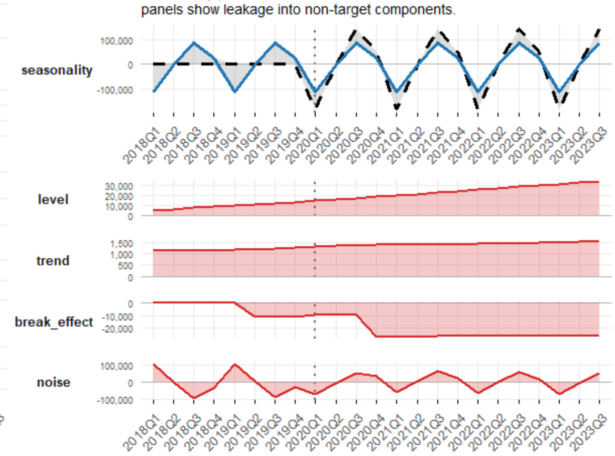
BSTS Decomposition of Perturbed Synthetic AY_3m Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic AY_3m series.



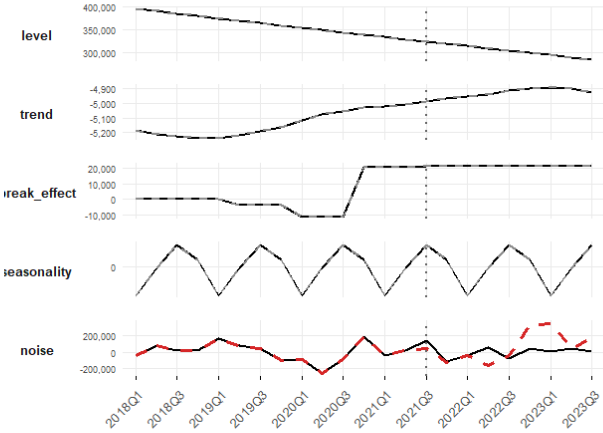
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (AY_3m | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



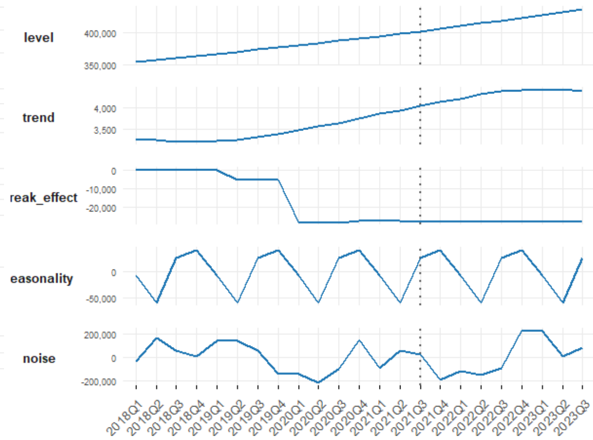
Ground-Truth Perturbation Decomposition (AY_3m | Stage_4_Noise: Noise Shock)

Baseline versus perturbed decomposition for AY_3m. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.



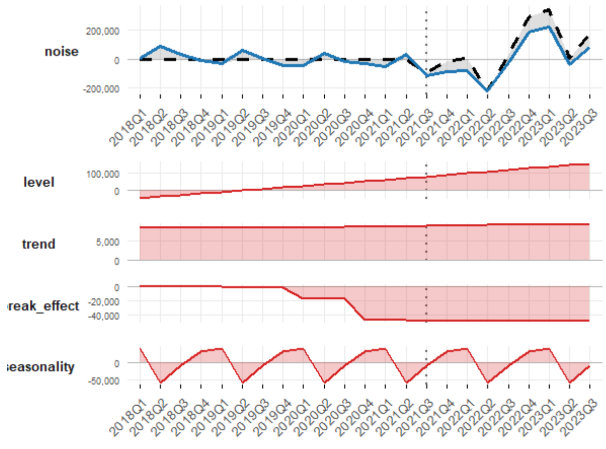
BSTS Decomposition of Perturbed Synthetic AY_3m Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic AY_3m series.



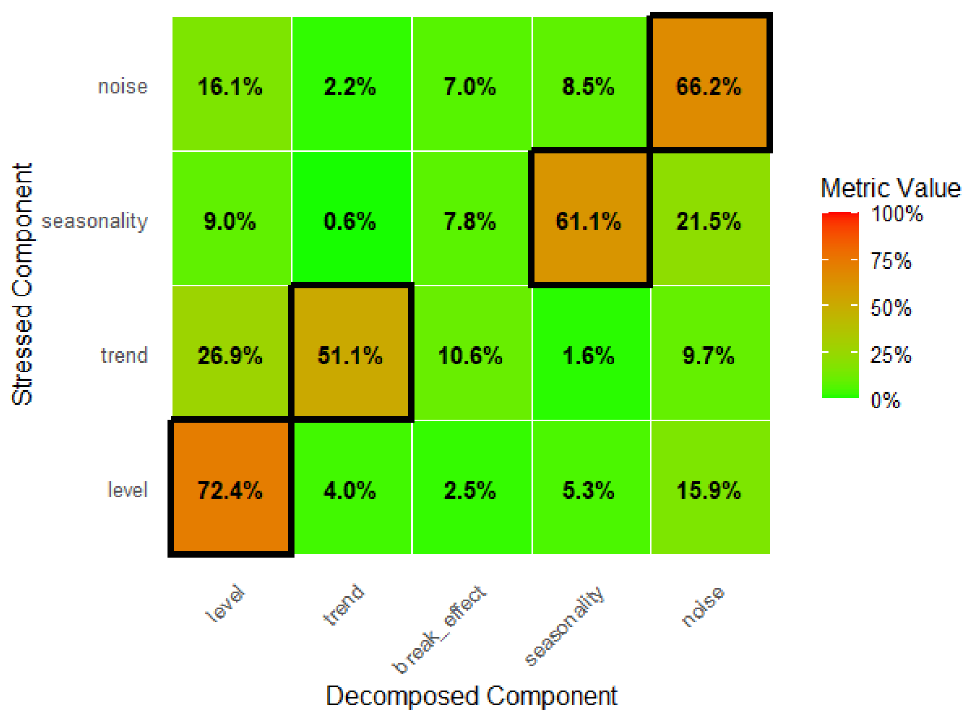
Difference Decomposition: Estimated Attribution versus True Noise Perturbation (AY_3m | Stage_4_Noise)

Noise panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



Recovery Error / Cross-Component Leakage Matrix (AY_3m)

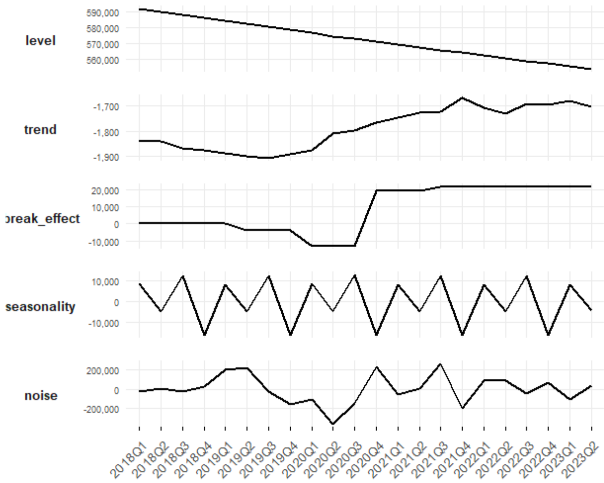
Rows are stressed components. Columns are decomposed components. Diagonal cells are Recovery Error; off-diagonal cells are Cross-Component Leakage.



AY (6 months):

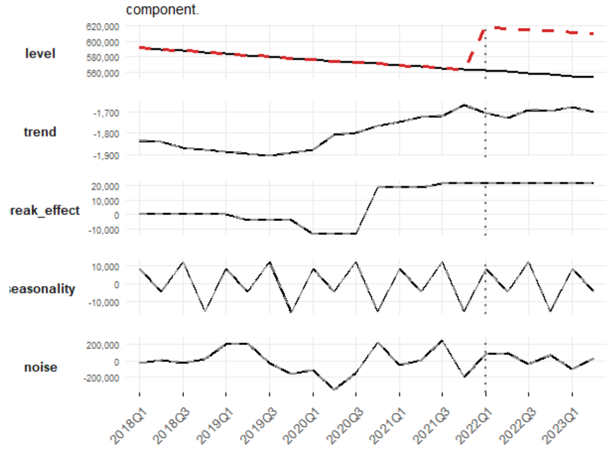
Figure 3.1 - Baseline BSTS Decomposition of AY_6m

Baseline BSTS decomposition of the AY_6m series.



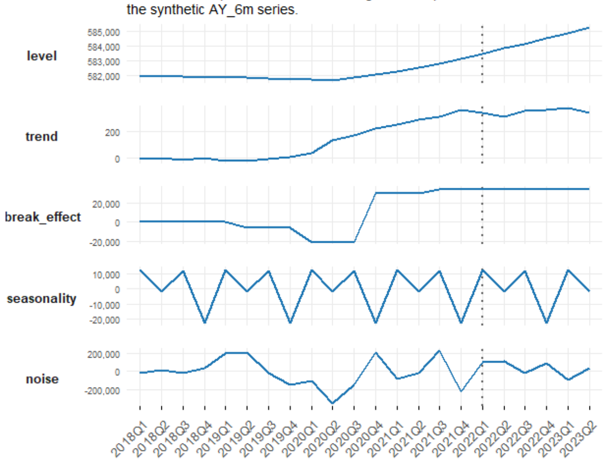
Ground-Truth Perturbation Decomposition (AY_6m | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for AY_6m. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



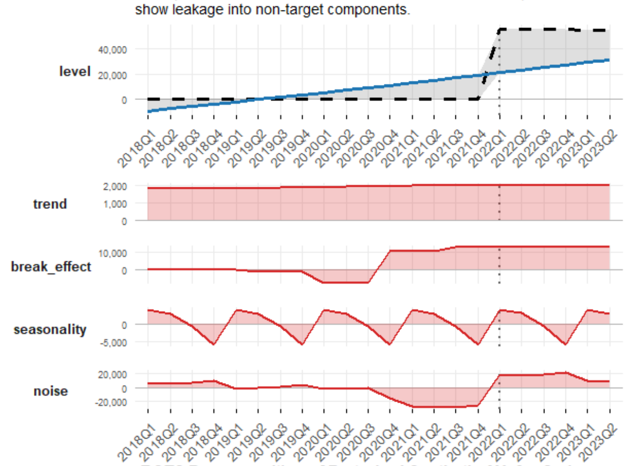
BSTS Decomposition of Perturbed Synthetic AY_6m Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic AY_6m series.



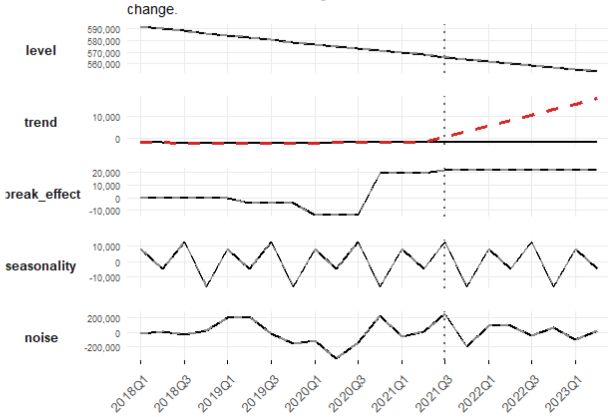
Difference Decomposition: Estimated Attribution versus True Level Perturbation (AY_6m | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



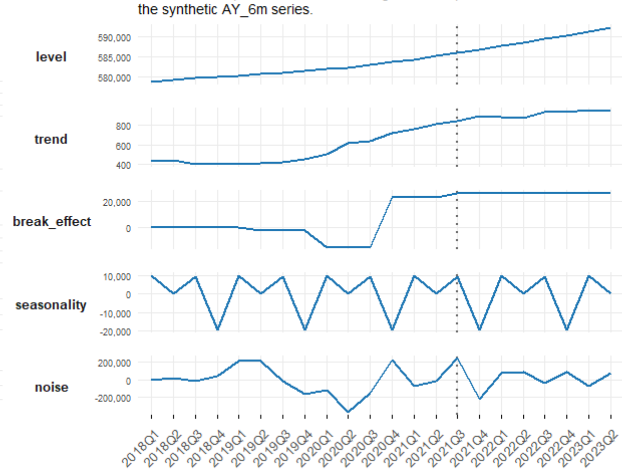
Ground-Truth Perturbation Decomposition (AY_6m | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for AY_6m. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



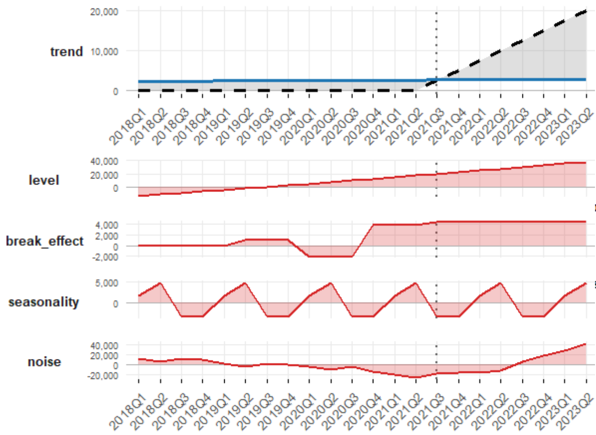
BSTS Decomposition of Perturbed Synthetic AY_6m Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic AY_6m series.



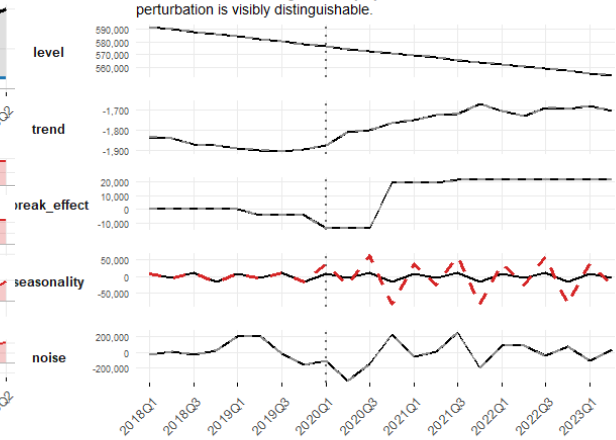
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (AY_6m | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



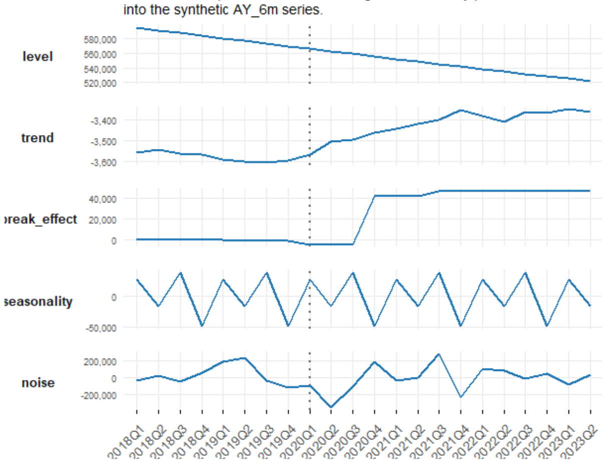
Ground-Truth Perturbation Decomposition (AY_6m | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for AY_6m. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



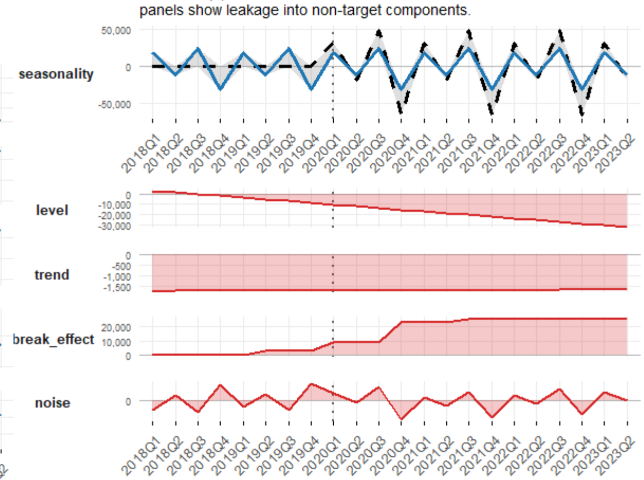
BSTS Decomposition of Perturbed Synthetic AY_6m Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic AY_6m series.



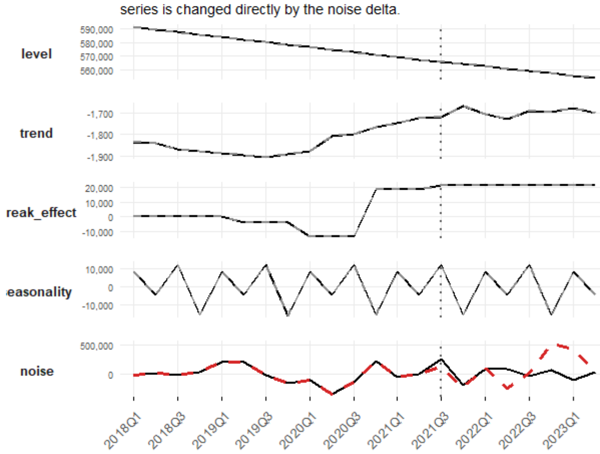
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (AY_6m | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



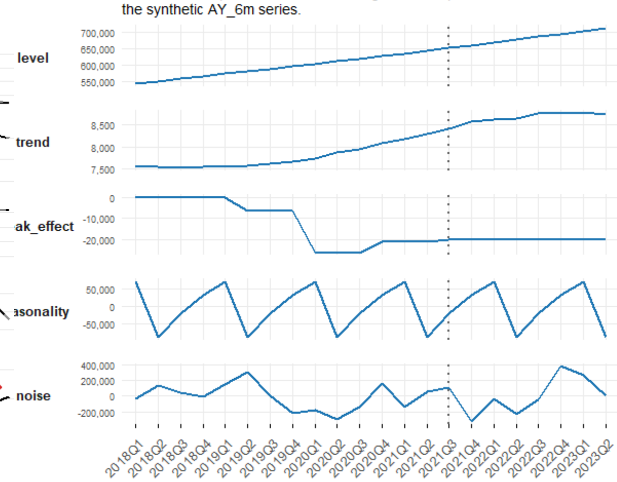
Ground-Truth Perturbation Decomposition (AY_6m | Stage_4_Noise: Noise Shock)

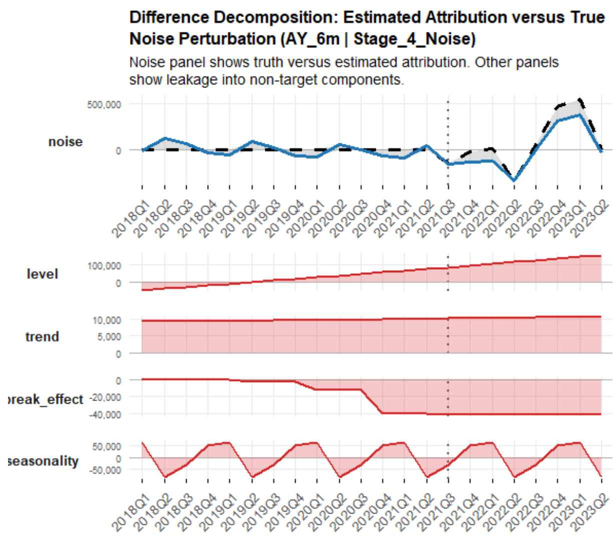
Baseline versus perturbed decomposition for AY_6m. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.

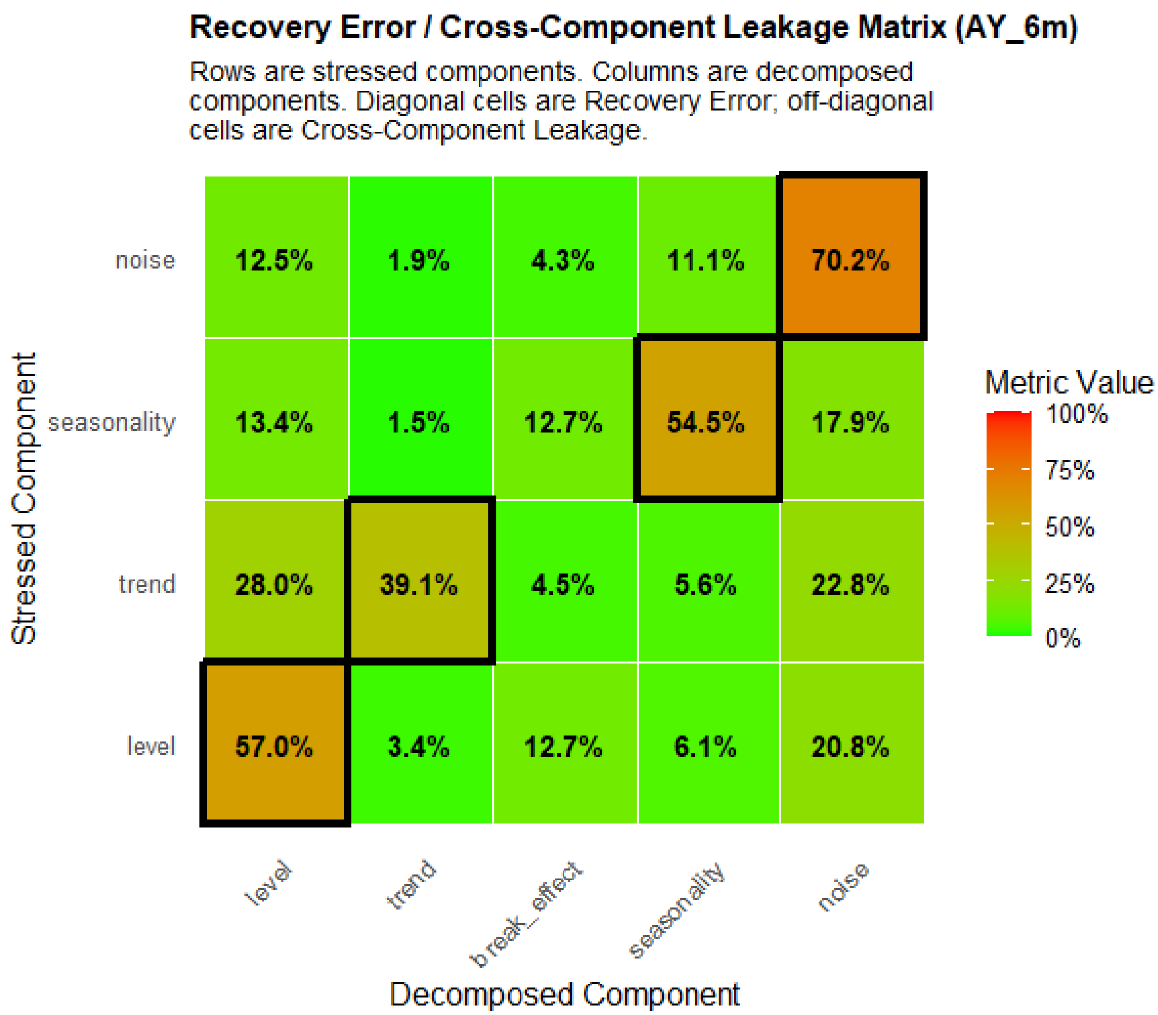


BSTS Decomposition of Perturbed Synthetic AY_6m Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic AY_6m series.



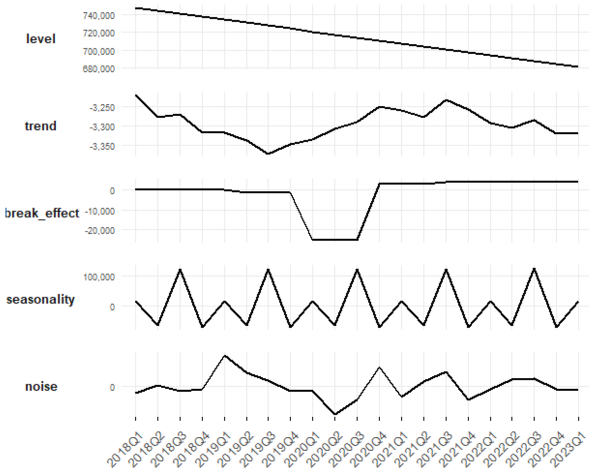

Difference Decomposition: Estimated Attribution versus True
Noise Perturbation (AY_6m | Stage_4_Noise)
Noise panel shows truth versus estimated attribution. Other panels
show leakage into non-target components.
noise
level
trend
break_effect
seasonality
500,000
0
100,000
10,000
5,000
-20,000
-40,000
50,000
-50,000
2018Q1
2018Q2
2018Q3
2018Q4
2019Q1
2019Q2
2019Q3
2019Q4
2020Q1
2020Q2
2020Q3
2020Q4
2021Q1
2021Q2
2021Q3
2021Q4
2022Q1
2022Q2
2022Q3
2022Q4
2023Q1
2023Q2


Recovery Error / Cross-Component Leakage Matrix (AY_6m)
Rows are stressed components. Columns are decomposed
components. Diagonal cells are Recovery Error; off-diagonal
cells are Cross-Component Leakage.
Stressed Component
noise
seasonality
trend
level
12.5%
1.9%
4.3%
11.1%
70.2%
13.4%
1.5%
12.7%
54.5%
17.9%
28.0%
39.1%
4.5%
5.6%
22.8%
57.0%
3.4%
12.7%
6.1%
20.8%
level
trend
break_effect
seasonality
noise
Decomposed Component
Metric Value
100%
75%
50%
25%
0%

AY (9 months):

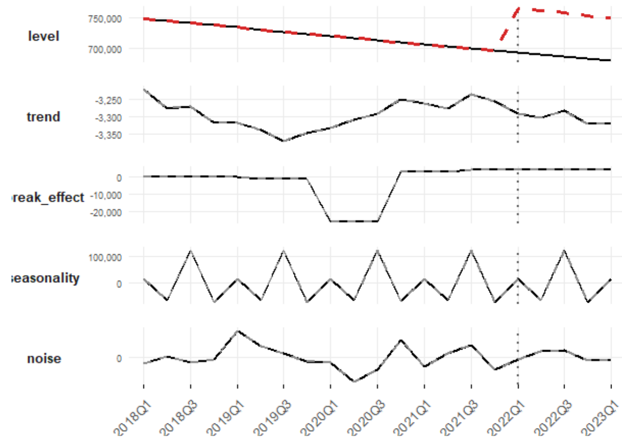
Figure 3.1 - Baseline BSTS Decomposition of AY_9m

Baseline BSTS decomposition of the AY_9m series.



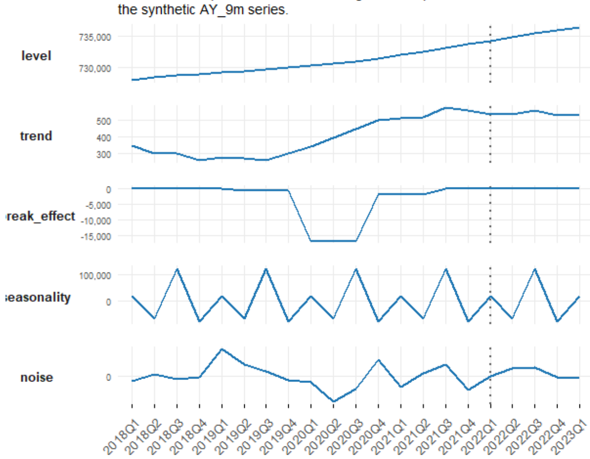
Ground-Truth Perturbation Decomposition (AY_9m | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for AY_9m. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



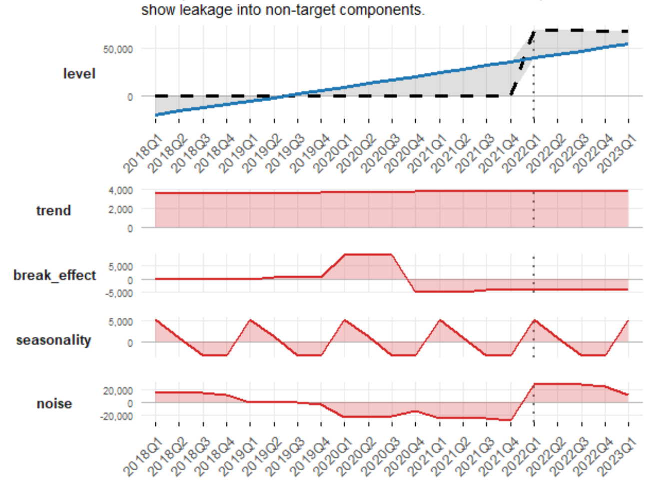
BSTS Decomposition of Perturbed Synthetic AY_9m Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic AY_9m series.



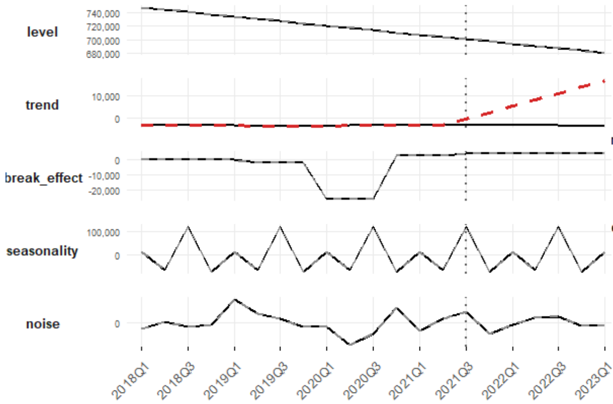
Difference Decomposition: Estimated Attribution versus True Level Perturbation (AY_9m | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



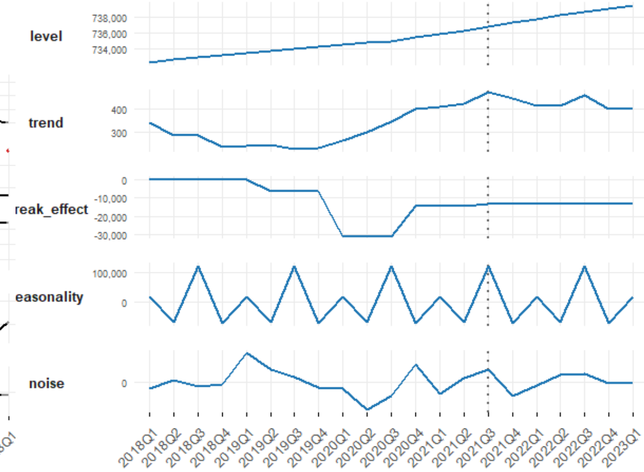
Ground-Truth Perturbation Decomposition (AY_9m | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for AY_9m. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



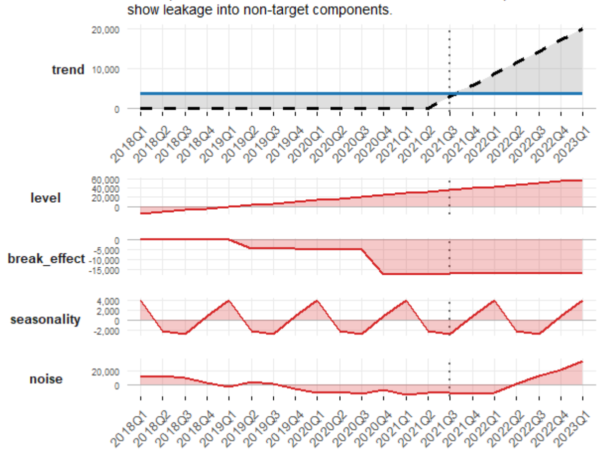
BSTS Decomposition of Perturbed Synthetic AY_9m Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic AY_9m series.



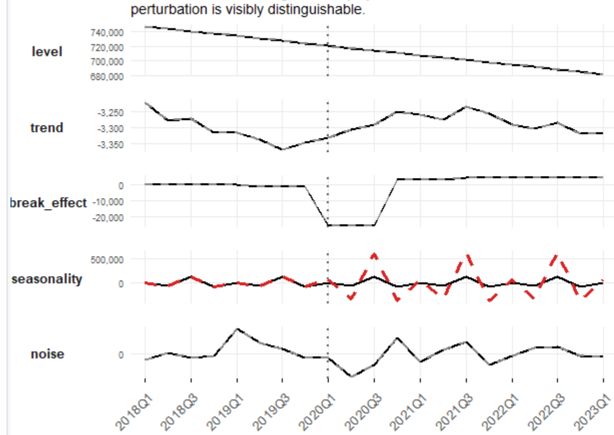
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (AY_9m | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



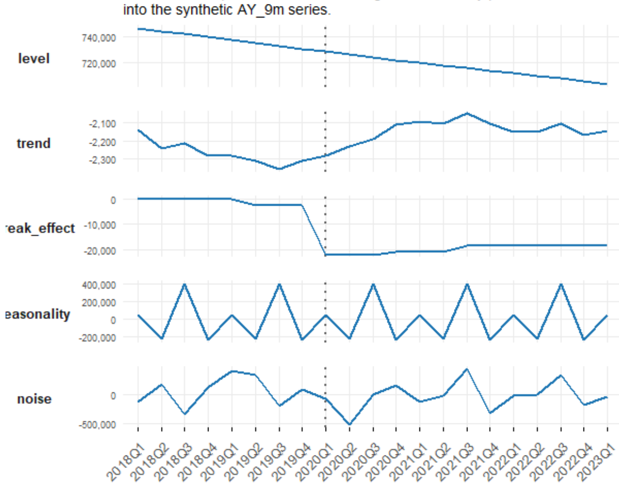
Ground-Truth Perturbation Decomposition (AY_9m | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for AY_9m. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



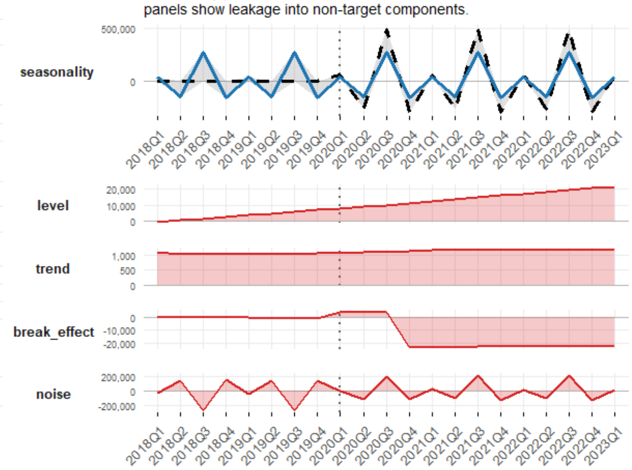
BSTS Decomposition of Perturbed Synthetic AY_9m Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic AY_9m series.



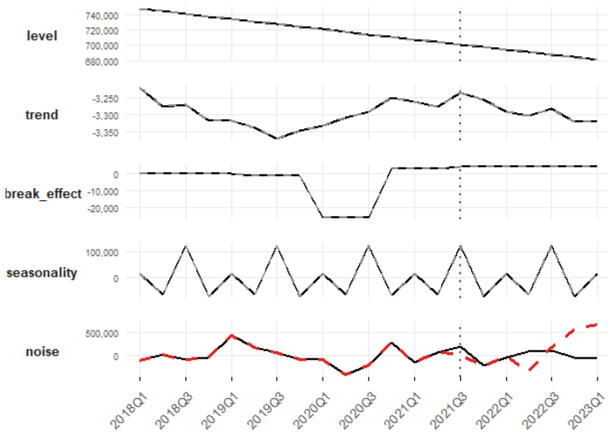
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (AY_9m | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



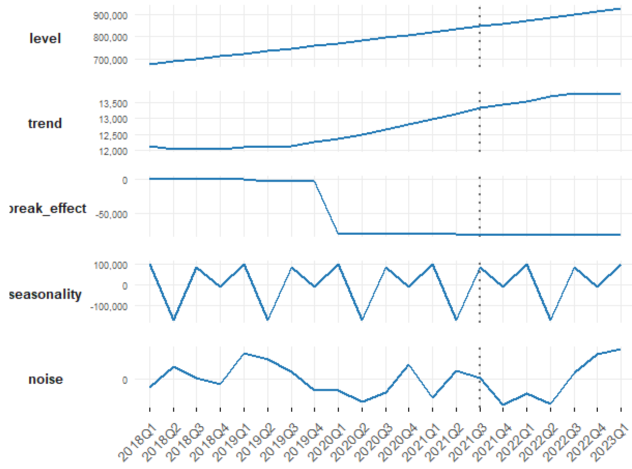
Ground-Truth Perturbation Decomposition (AY_9m | Stage_4_Noise: Noise Shock)

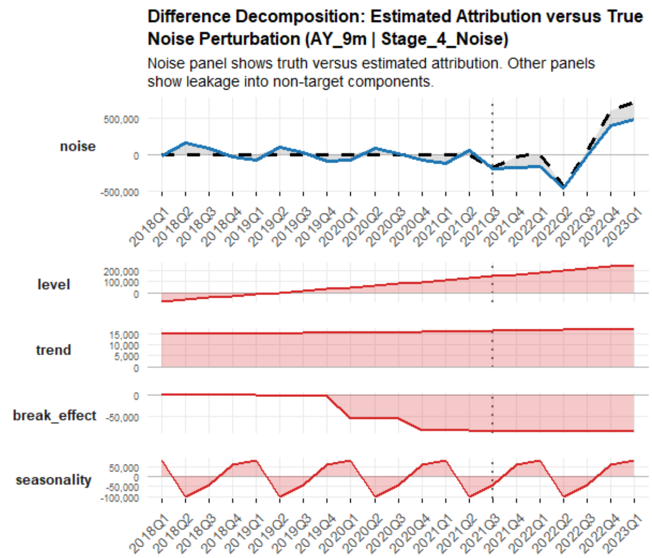
Baseline versus perturbed decomposition for AY_9m. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.

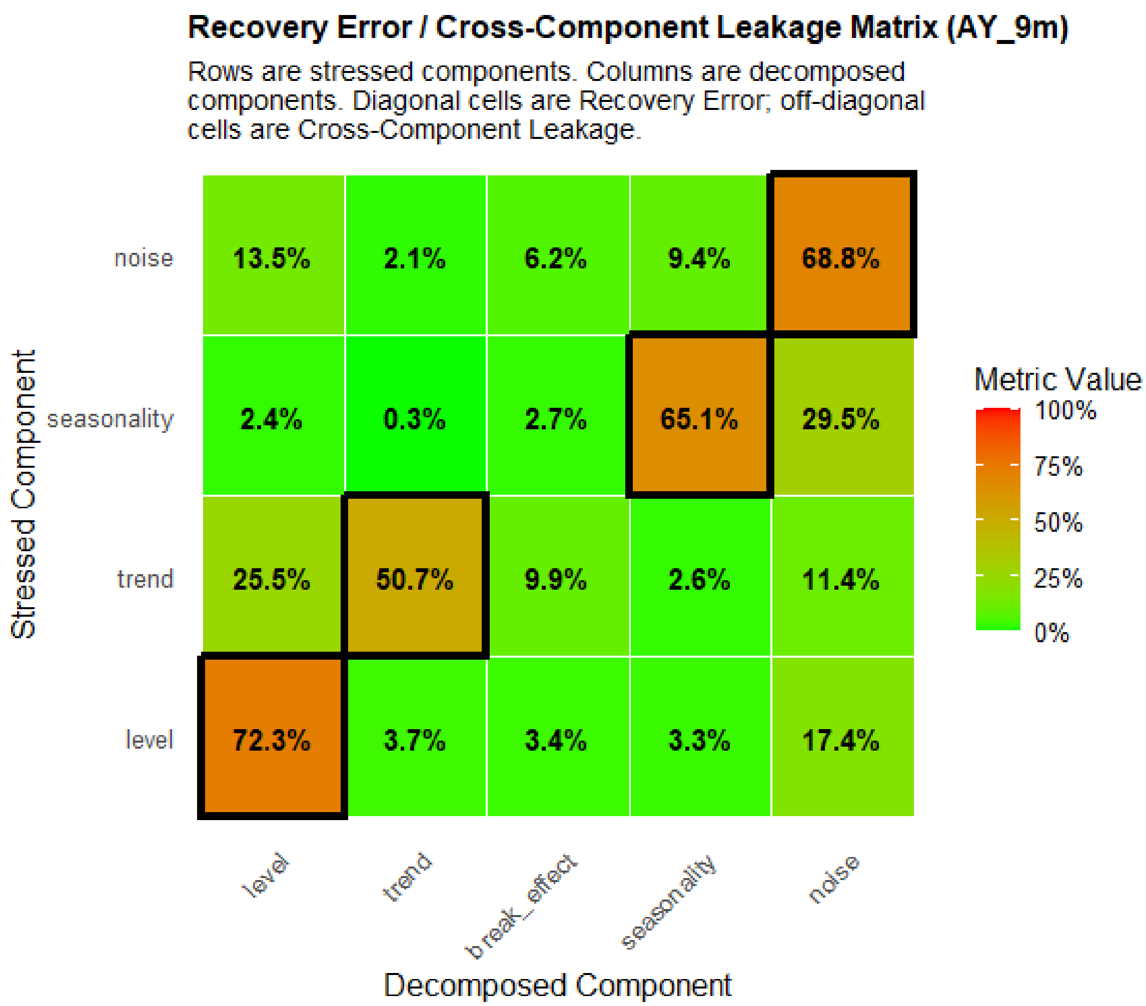


BSTS Decomposition of Perturbed Synthetic AY_9m Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic AY_9m series.



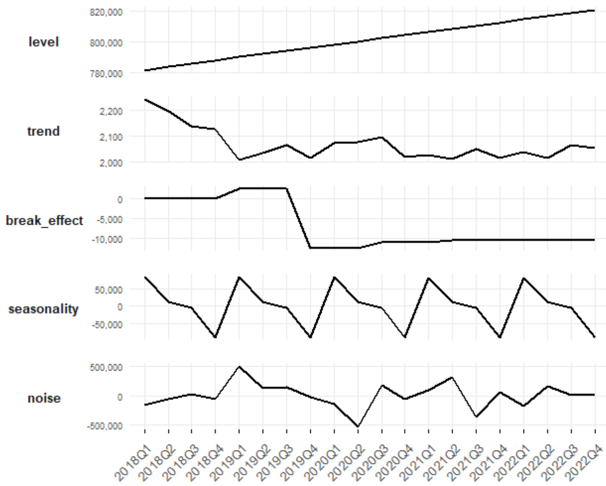

Difference Decomposition: Estimated Attribution versus True Noise Perturbation (AY_9m | Stage_4_Noise)
Noise panel shows truth versus estimated attribution. Other panels show leakage into non-target components.
noise
level
trend
break_effect
seasonality
2018Q1 2018Q2 2018Q3 2018Q4 2019Q1 2019Q2 2019Q3 2019Q4 2020Q1 2020Q2 2020Q3 2020Q4 2021Q1 2021Q2 2021Q3 2021Q4 2022Q1 2022Q2 2022Q3 2022Q4 2023Q1


Recovery Error / Cross-Component Leakage Matrix (AY_9m)
Rows are stressed components. Columns are decomposed components. Diagonal cells are Recovery Error; off-diagonal cells are Cross-Component Leakage.
Stressed Component
Decomposed Component
noise: 13.5% 2.1% 6.2% 9.4% 68.8%
seasonality: 2.4% 0.3% 2.7% 65.1% 29.5%
trend: 25.5% 50.7% 9.9% 2.6% 11.4%
level: 72.3% 3.7% 3.4% 3.3% 17.4%
level trend break_effect seasonality noise
Metric Value
100%
75%
50%
25%
0%

AY (12 months):

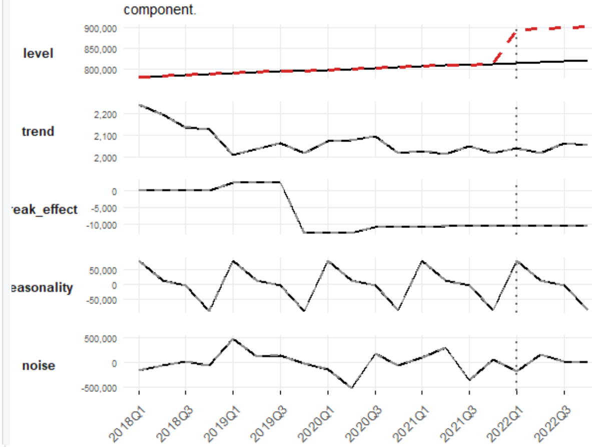
Figure 3.1 - Baseline BSTS Decomposition of AY_12m

Baseline BSTS decomposition of the AY_12m series.



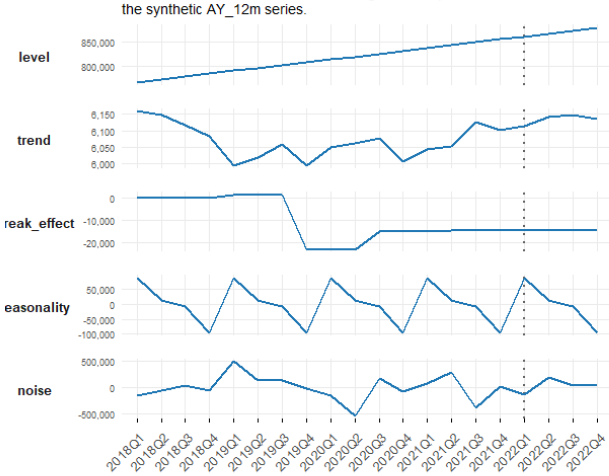
Ground-Truth Perturbation Decomposition (AY_12m | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for AY_12m. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



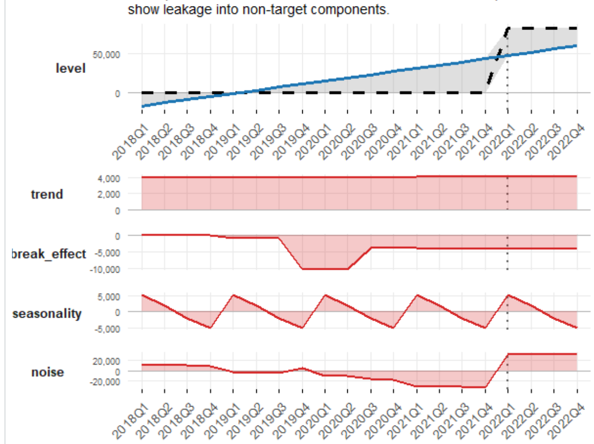
BSTS Decomposition of Perturbed Synthetic AY_12m Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic AY_12m series.



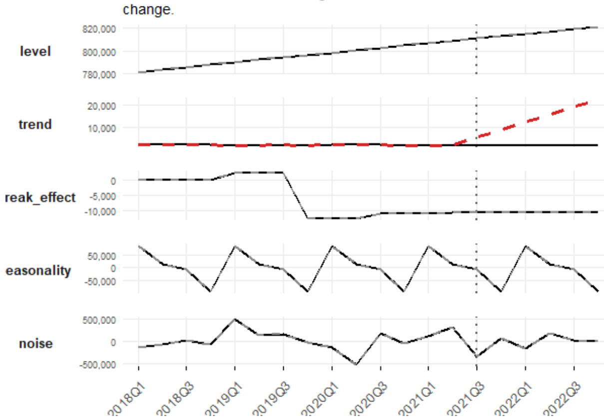
Difference Decomposition: Estimated Attribution versus True Level Perturbation (AY_12m | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



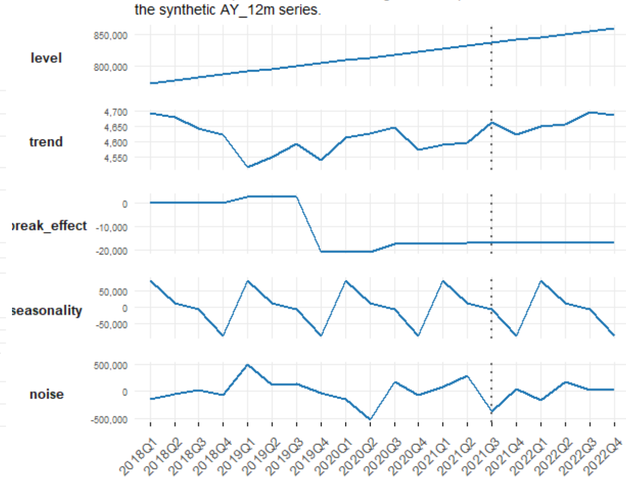
Ground-Truth Perturbation Decomposition (AY_12m | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for AY_12m. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



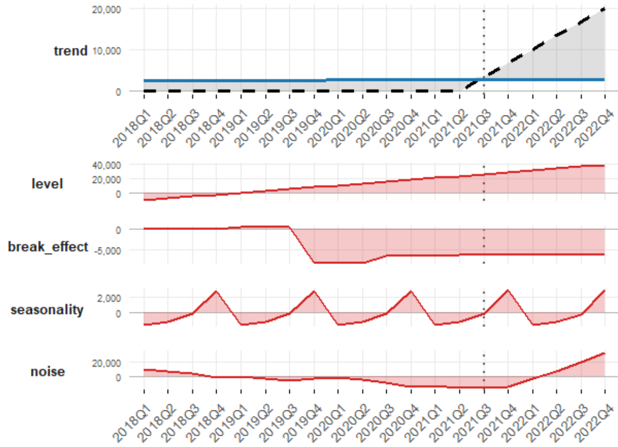
BSTS Decomposition of Perturbed Synthetic AY_12m Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic AY_12m series.



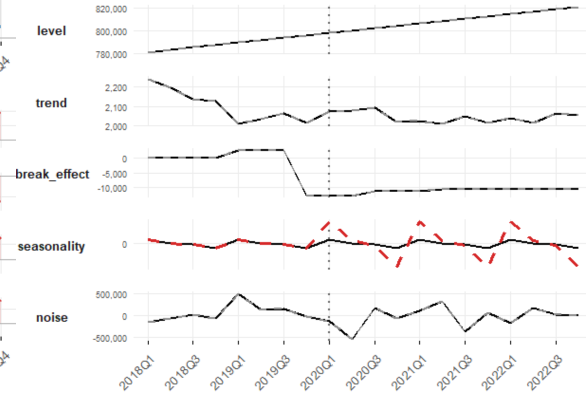
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (AY_12m | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



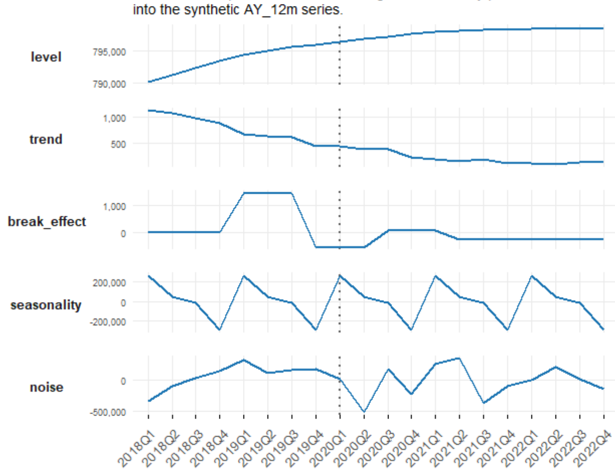
Ground-Truth Perturbation Decomposition (AY_12m | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for AY_12m. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



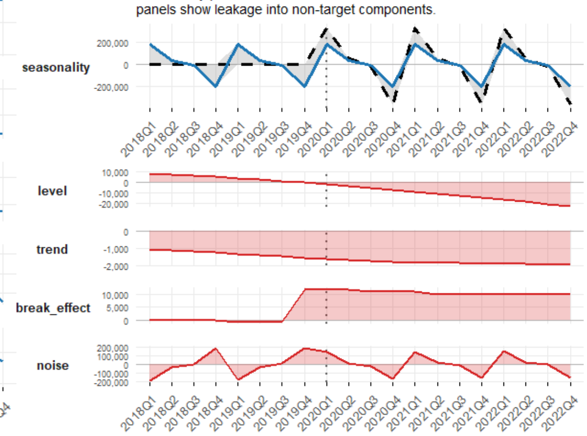
BSTS Decomposition of Perturbed Synthetic AY_12m Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic AY_12m series.



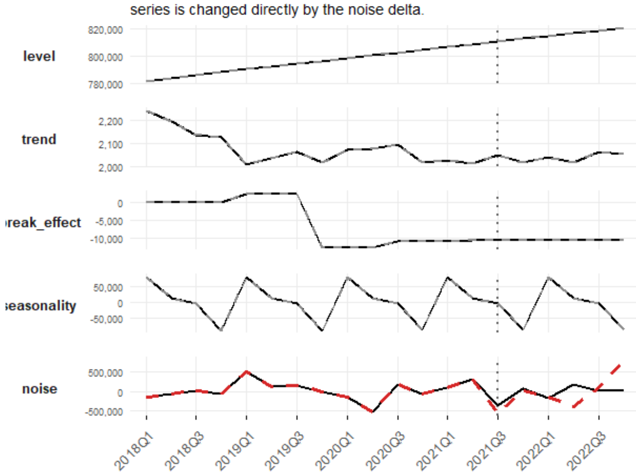
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (AY_12m | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



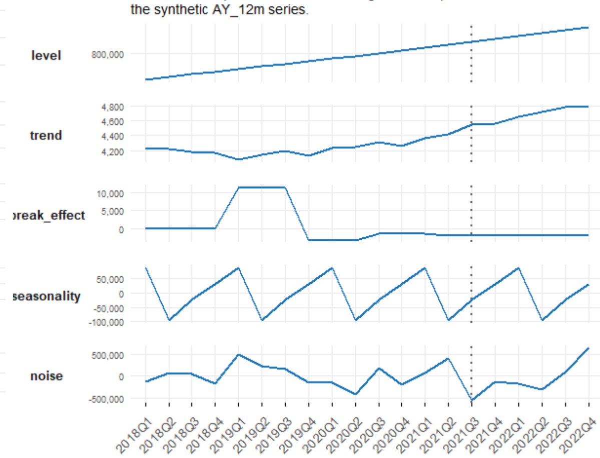
Ground-Truth Perturbation Decomposition (AY_12m | Stage_4_Noise: Noise Shock)

Baseline versus perturbed decomposition for AY_12m. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.



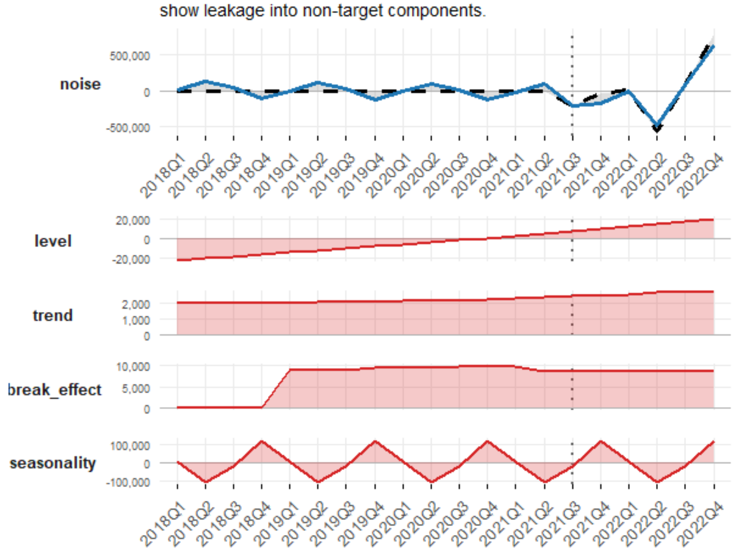
BSTS Decomposition of Perturbed Synthetic AY_12m Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic AY_12m series.



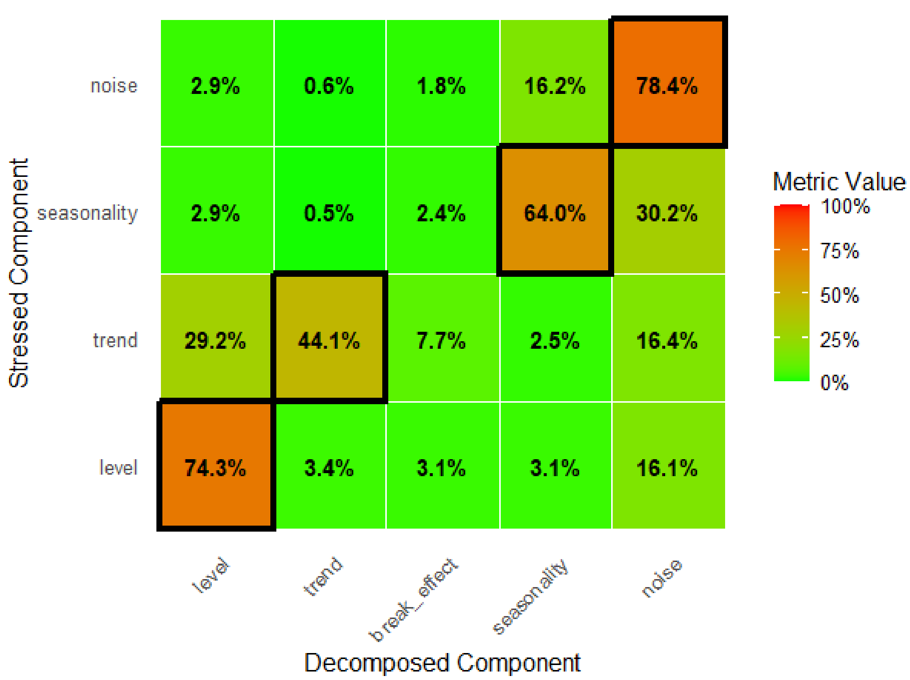
Difference Decomposition: Estimated Attribution versus True Noise Perturbation (AY_12m | Stage_4_Noise)

Noise panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



Recovery Error / Cross-Component Leakage Matrix (AY_12m)

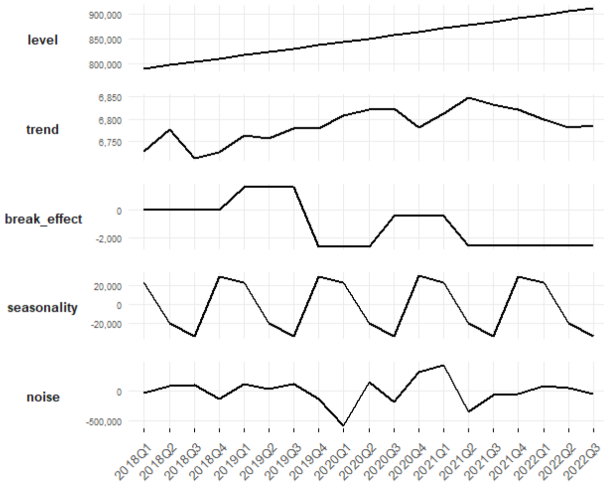
Rows are stressed components. Columns are decomposed components. Diagonal cells are Recovery Error; off-diagonal cells are Cross-Component Leakage.



AY (15 months):

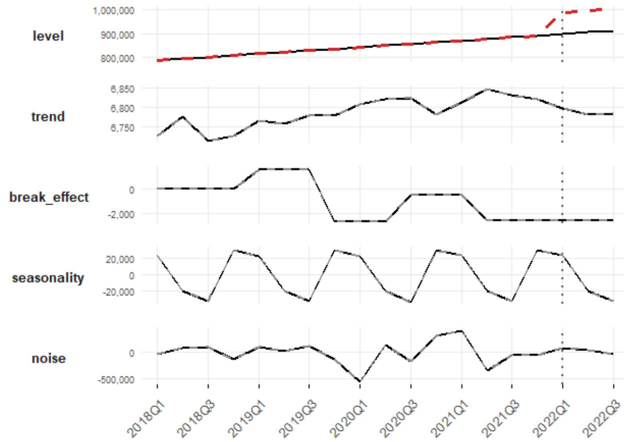
Figure 3.1 - Baseline BSTS Decomposition of AY_15m

Baseline BSTS decomposition of the AY_15m series.



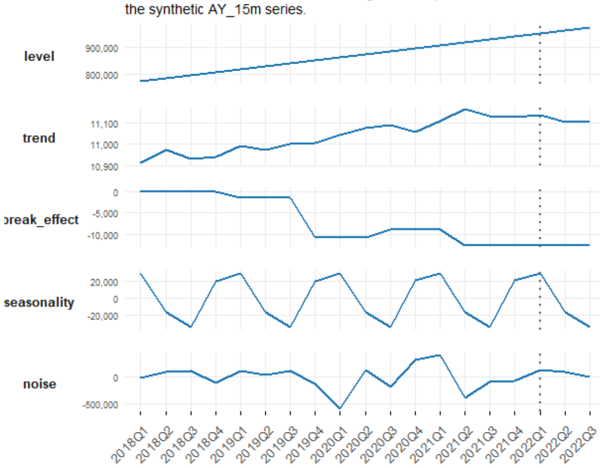
Ground-Truth Perturbation Decomposition (AY_15m | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for AY_15m. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



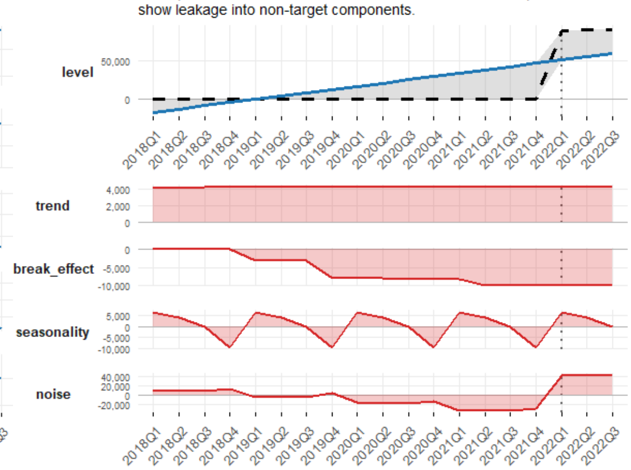
BSTS Decomposition of Perturbed Synthetic AY_15m Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic AY_15m series.



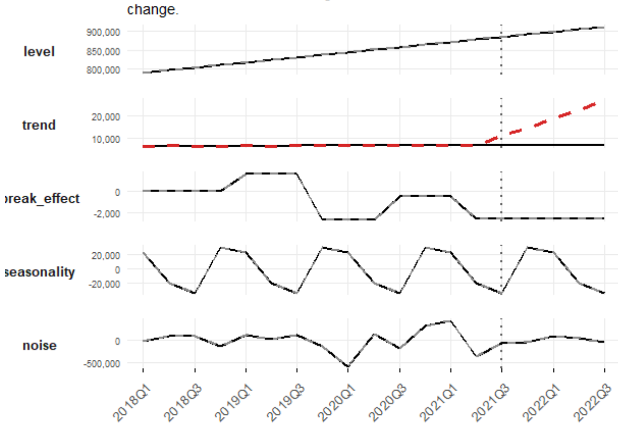
Difference Decomposition: Estimated Attribution versus True Level Perturbation (AY_15m | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



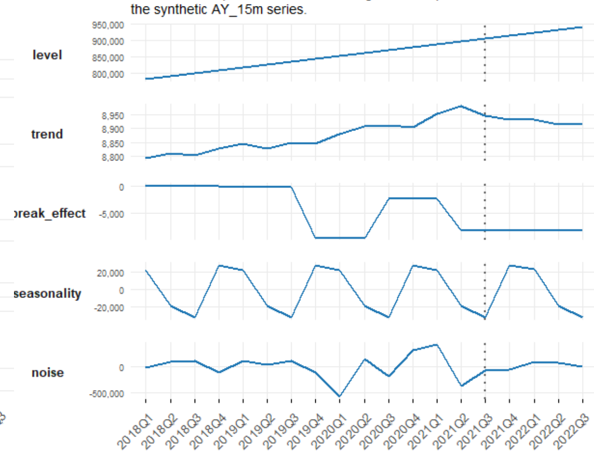
Ground-Truth Perturbation Decomposition (AY_15m | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for AY_15m. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



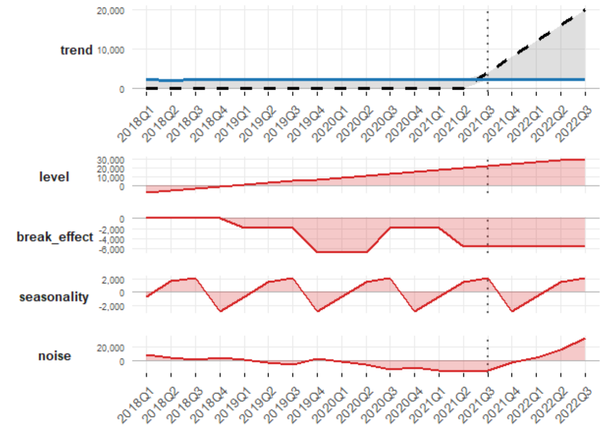
BSTS Decomposition of Perturbed Synthetic AY_15m Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic AY_15m series.



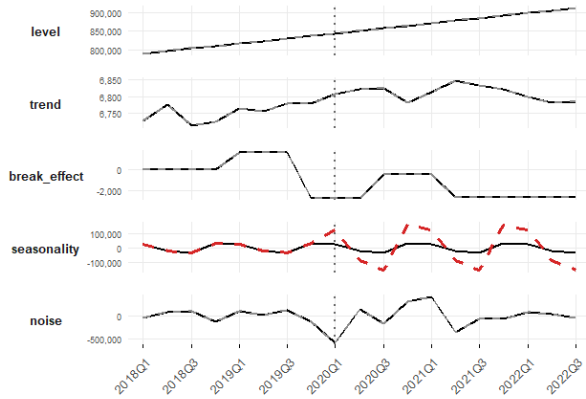
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (AY_15m | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



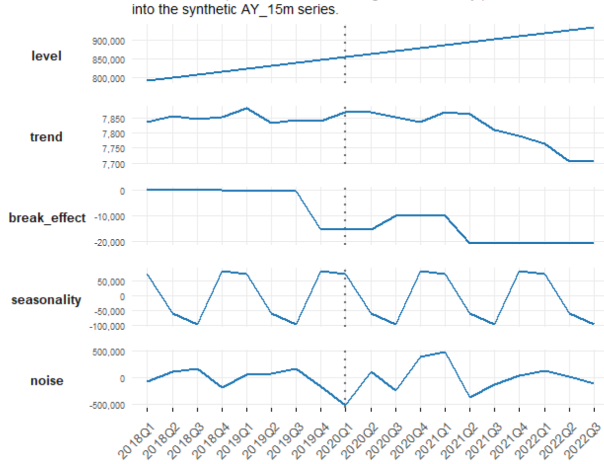
Ground-Truth Perturbation Decomposition (AY_15m | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for AY_15m. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



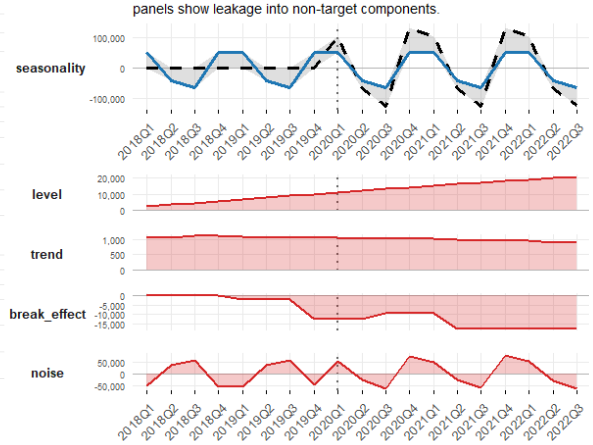
BSTS Decomposition of Perturbed Synthetic AY_15m Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic AY_15m series.



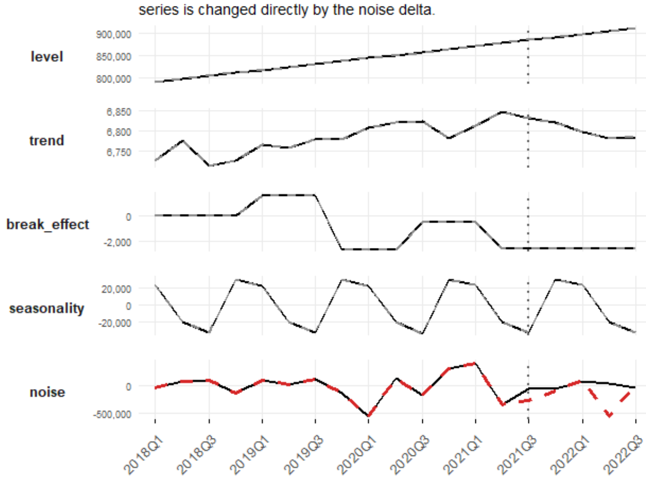
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (AY_15m | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



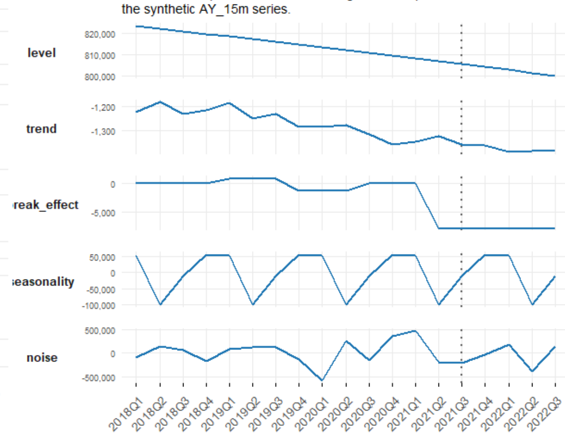
Ground-Truth Perturbation Decomposition (AY_15m | Stage_4_Noise: Noise Shock)

Baseline versus perturbed decomposition for AY_15m. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.



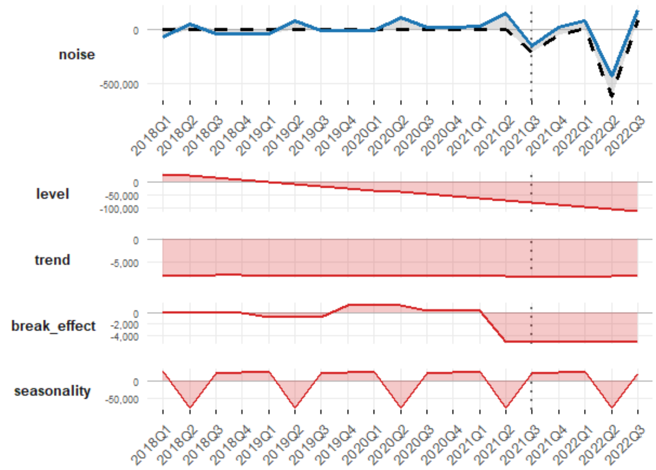
BSTS Decomposition of Perturbed Synthetic AY_15m Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic AY_15m series.



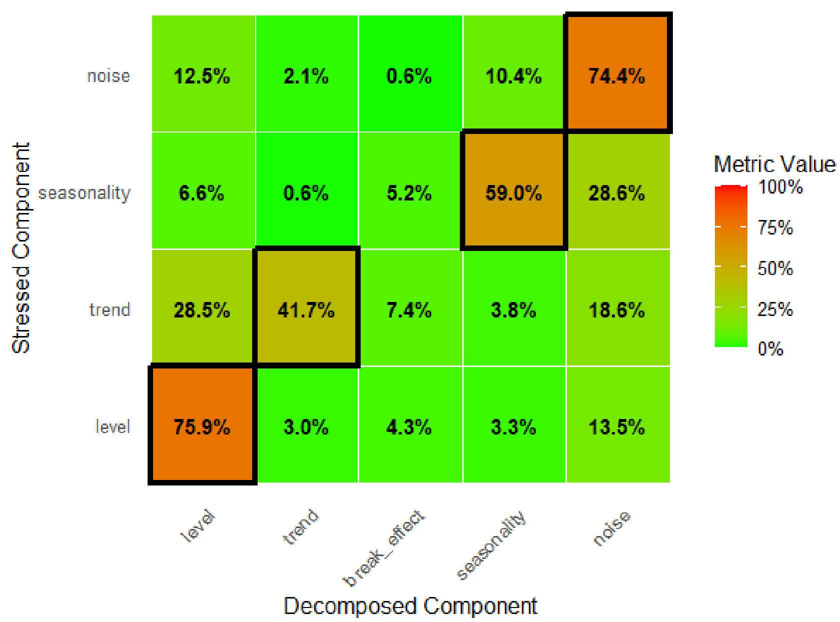
Difference Decomposition: Estimated Attribution versus True Noise Perturbation (AY_15m | Stage_4_Noise)

Noise panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



Recovery Error / Cross-Component Leakage Matrix (AY_15m)

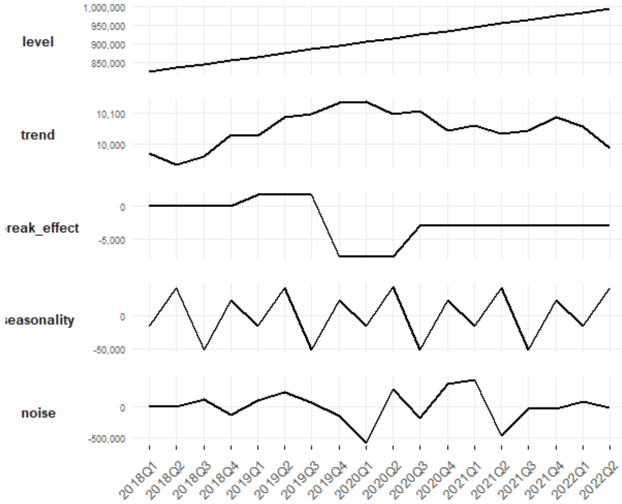
Rows are stressed components. Columns are decomposed components. Diagonal cells are Recovery Error; off-diagonal cells are Cross-Component Leakage.



AY (18 months):

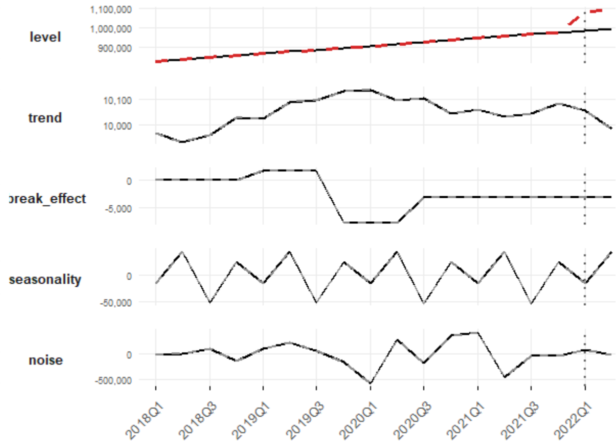
Figure 3.1 - Baseline BSTS Decomposition of AY_18m

Baseline BSTS decomposition of the AY_18m series.



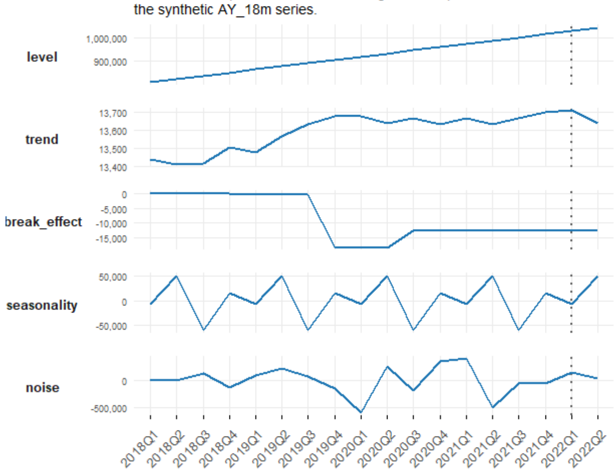
Ground-Truth Perturbation Decomposition (AY_18m | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for AY_18m. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



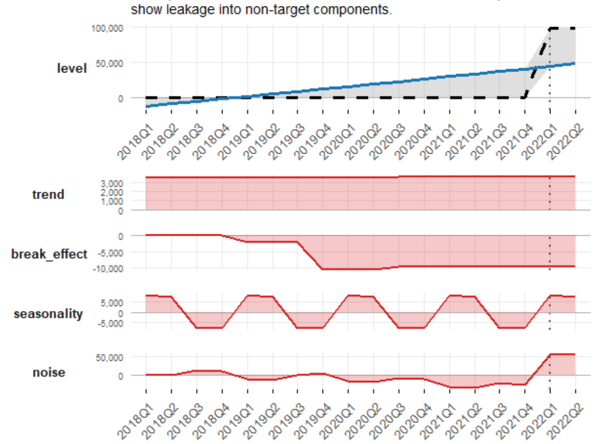
BSTS Decomposition of Perturbed Synthetic AY_18m Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic AY_18m series.



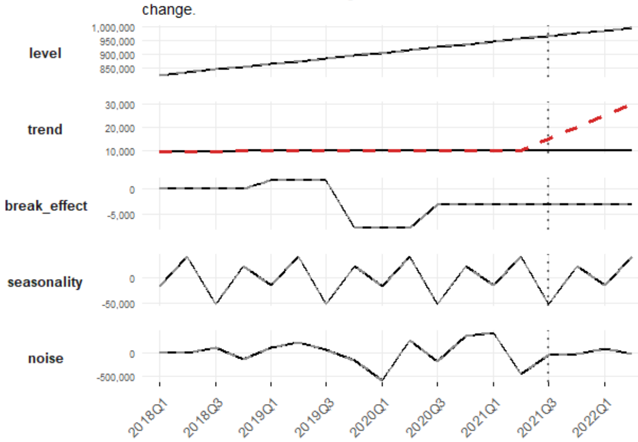
Difference Decomposition: Estimated Attribution versus True Level Perturbation (AY_18m | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



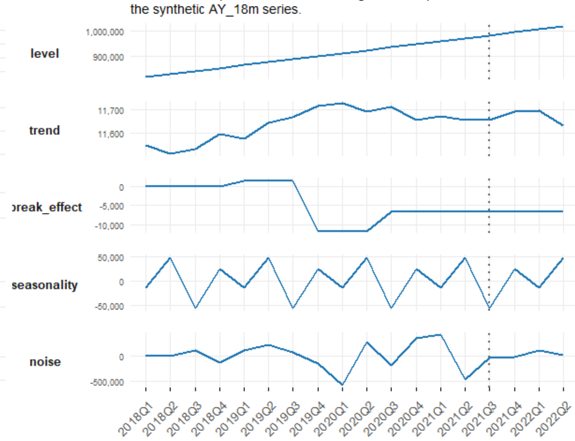
Ground-Truth Perturbation Decomposition (AY_18m | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for AY_18m. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



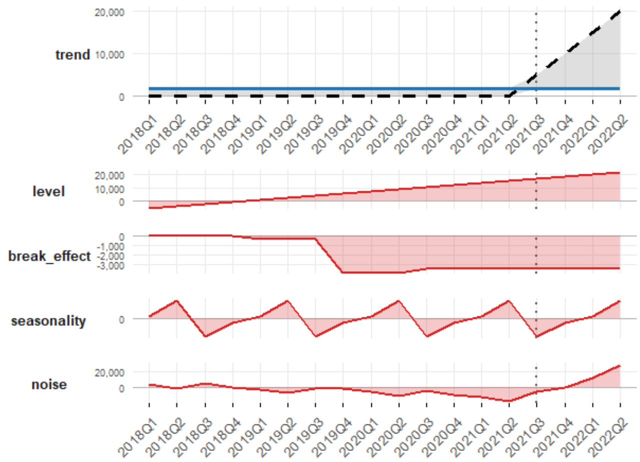
BSTS Decomposition of Perturbed Synthetic AY_18m Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic AY_18m series.



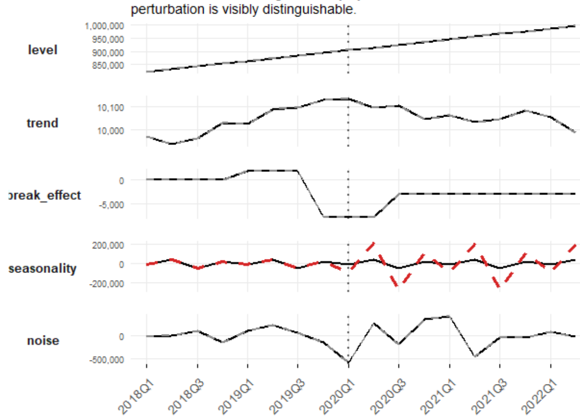
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (AY_18m | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



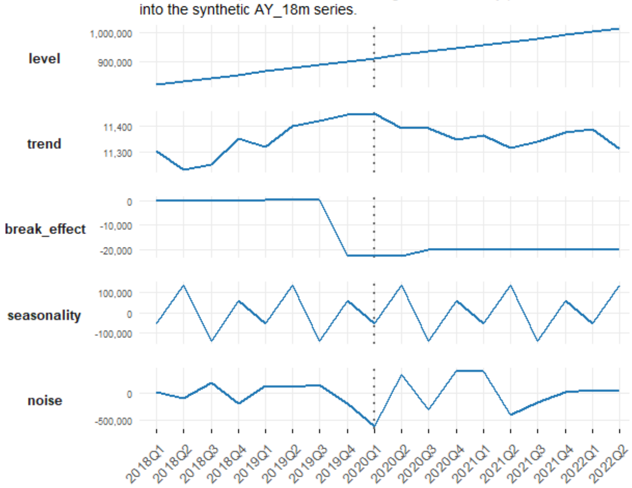
Ground-Truth Perturbation Decomposition (AY_18m | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for AY_18m. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



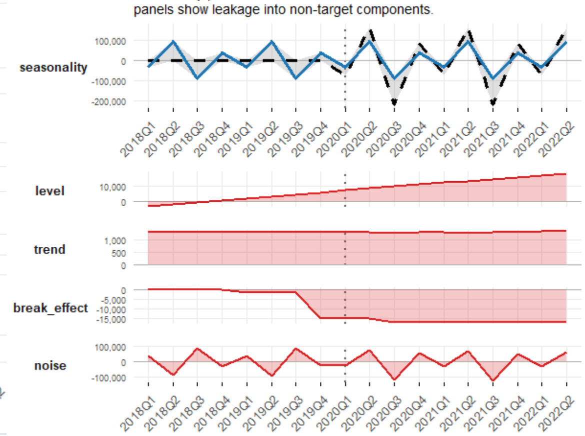
BSTS Decomposition of Perturbed Synthetic AY_18m Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic AY_18m series.



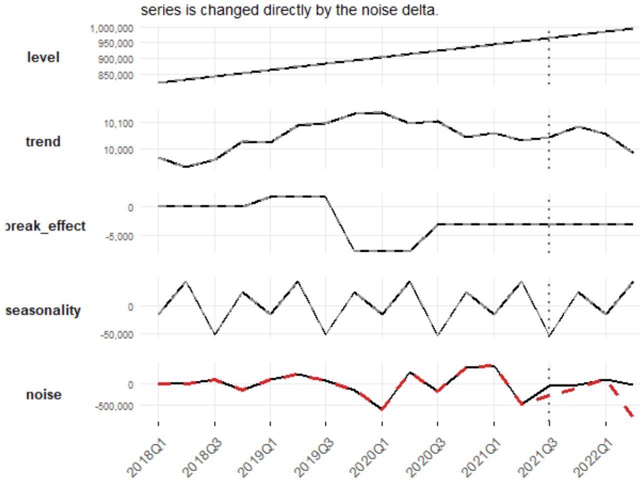
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (AY_18m | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



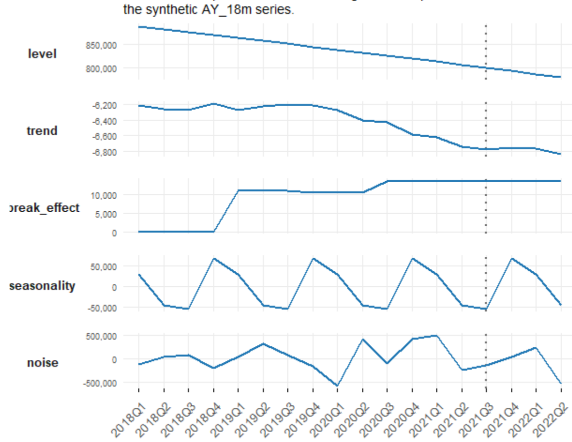
Ground-Truth Perturbation Decomposition (AY_18m | Stage_4_Noise: Noise Shock)

Baseline versus perturbed decomposition for AY_18m. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.



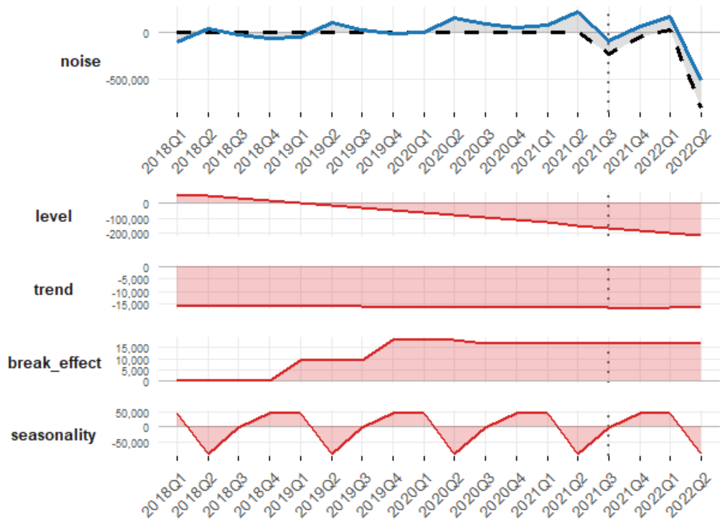
BSTS Decomposition of Perturbed Synthetic AY_18m Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic AY_18m series.



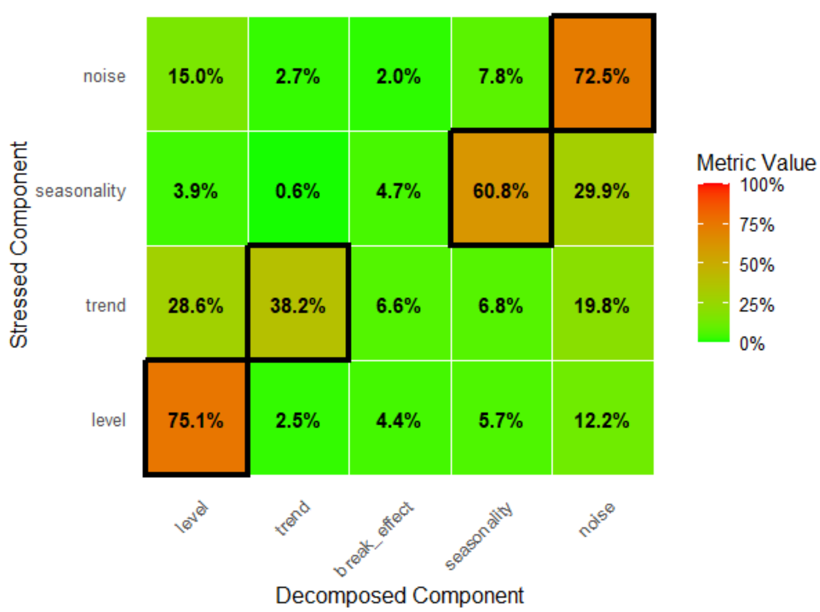
Difference Decomposition: Estimated Attribution versus True Noise Perturbation (AY_18m | Stage_4_Noise)

Noise panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



Recovery Error / Cross-Component Leakage Matrix (AY_18m)

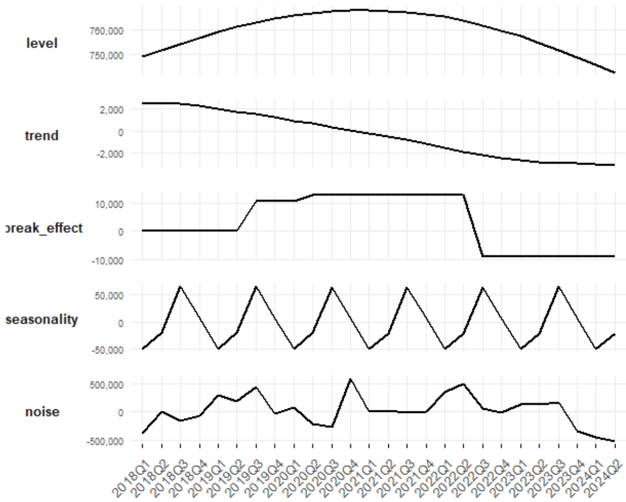
Rows are stressed components. Columns are decomposed components. Diagonal cells are Recovery Error; off-diagonal cells are Cross-Component Leakage.



CY:

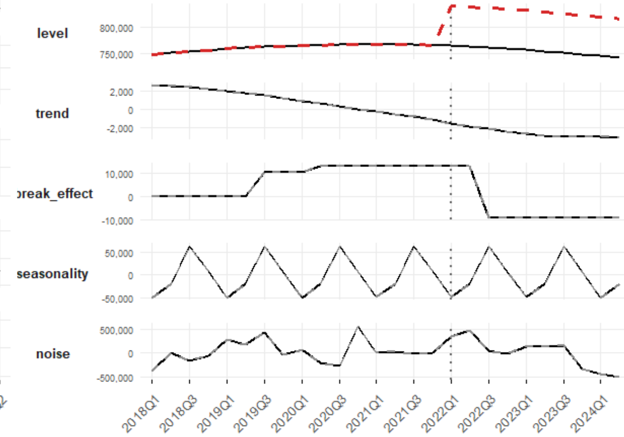
Figure 3.1 - Baseline BSTS Decomposition of CY

Baseline BSTS decomposition of the CY series.



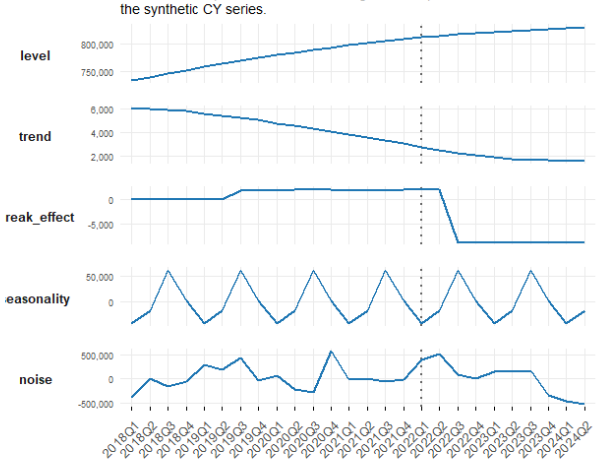
Ground-Truth Perturbation Decomposition (CY | Stage_1_Level: Level Shock)

Baseline versus perturbed decomposition for CY. Level is increased by 10% beginning at 2022Q1. The red line highlights the altered level component.



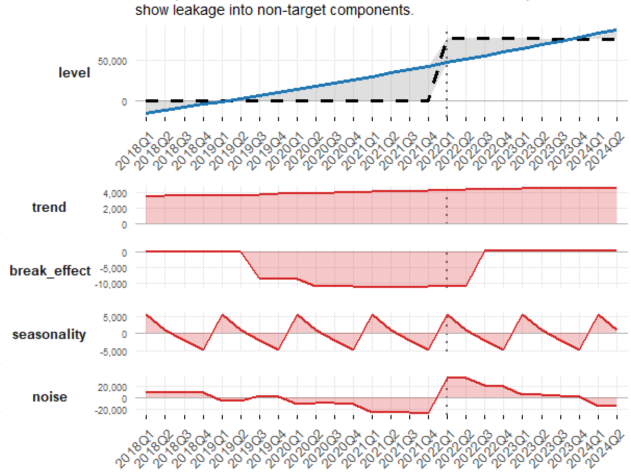
BSTS Decomposition of Perturbed Synthetic CY Series (Stage_1_Level)

Refitted decomposition after introducing the level perturbation into the synthetic CY series.



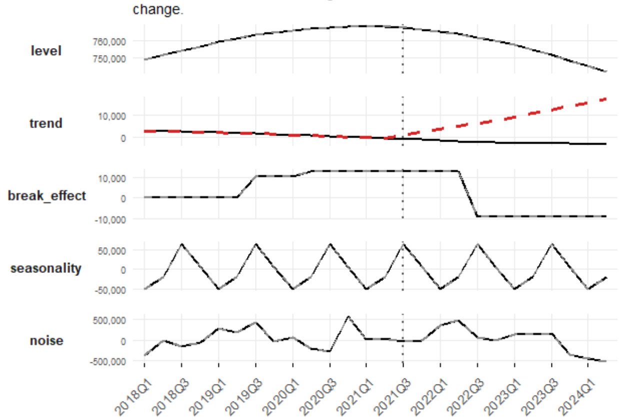
Difference Decomposition: Estimated Attribution versus True Level Perturbation (CY | Stage_1_Level)

Level panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



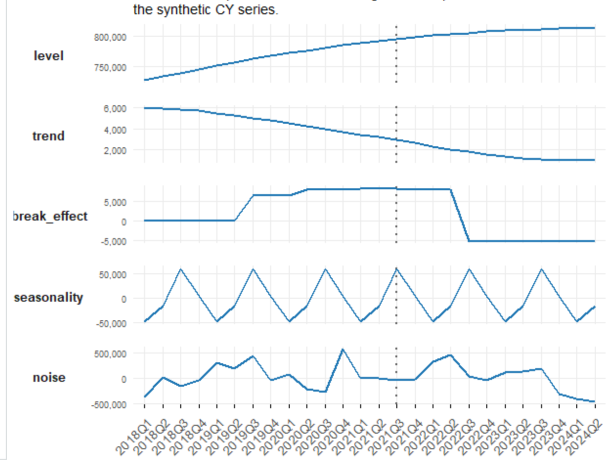
Ground-Truth Perturbation Decomposition (CY | Stage_2_Trend: Trend Ramp)

Baseline versus perturbed decomposition for CY. ONLY the trend component is perturbed beginning at 2021Q3. The level component is intentionally left unchanged in the truth decomposition. The synthetic observed series is altered using the cumulative effect of the trend change.



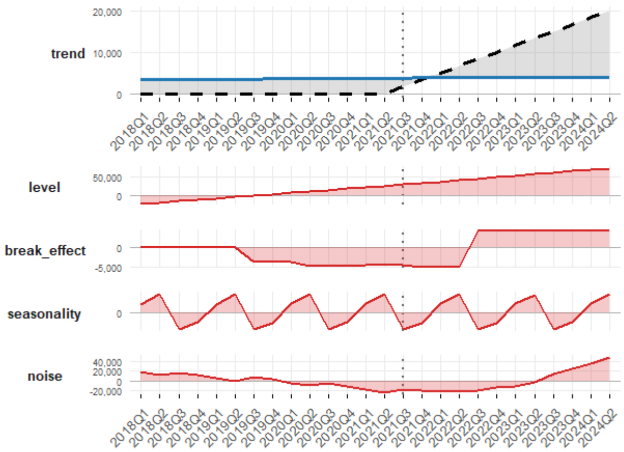
BSTS Decomposition of Perturbed Synthetic CY Series (Stage_2_Trend)

Refitted decomposition after introducing the trend perturbation into the synthetic CY series.



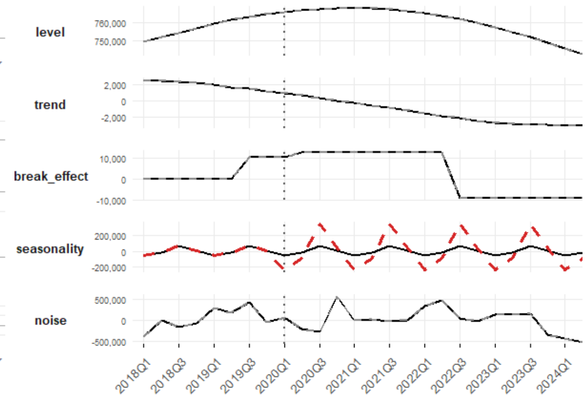
Difference Decomposition: Estimated Attribution versus True Trend Perturbation (CY | Stage_2_Trend)

Trend panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



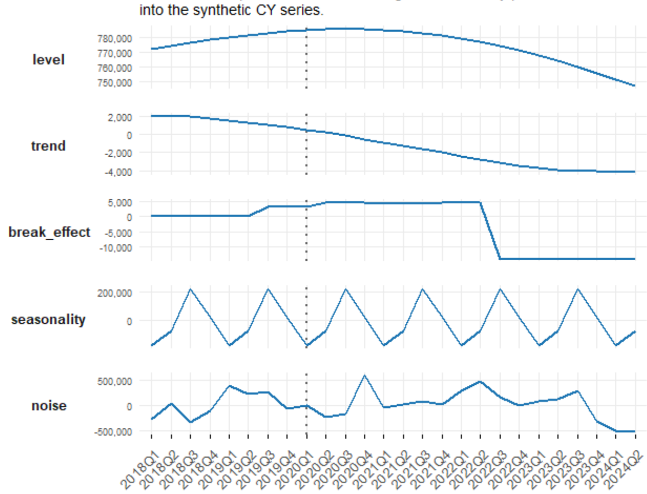
Ground-Truth Perturbation Decomposition (CY | Stage_3_Seasonality: Seasonality Amplification)

Baseline versus perturbed decomposition for CY. ONLY the seasonality component is amplified beginning at 2020Q1. The synthetic observed series is changed directly by the seasonal delta so the perturbation is visibly distinguishable.



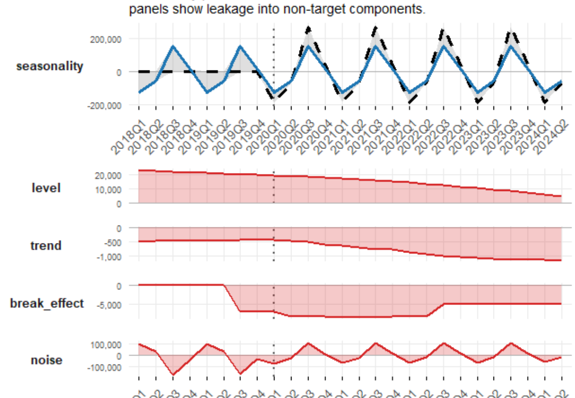
BSTS Decomposition of Perturbed Synthetic CY Series (Stage_3_Seasonality)

Refitted decomposition after introducing the seasonality perturbation into the synthetic CY series.



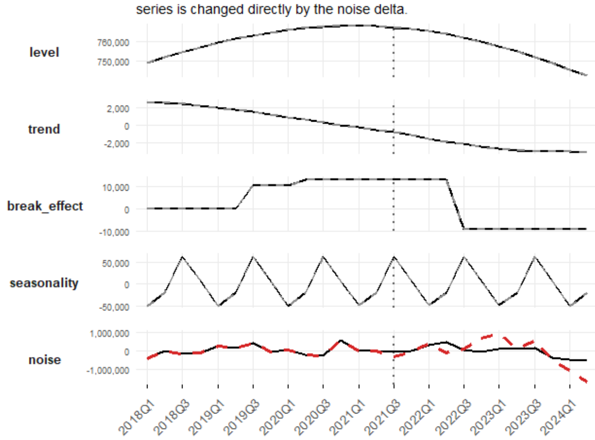
Difference Decomposition: Estimated Attribution versus True Seasonality Perturbation (CY | Stage_3_Seasonality)

Seasonality panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



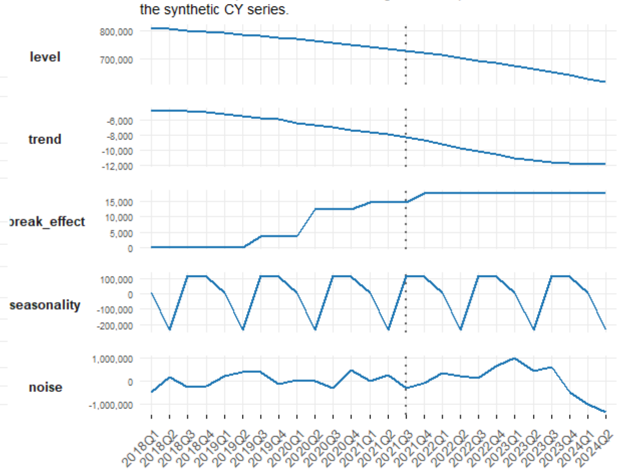
Ground-Truth Perturbation Decomposition (CY | Stage_4_Noise Noise Shock)

Baseline versus perturbed decomposition for CY. ONLY the noise component is shocked beginning at 2021Q3. The synthetic observed series is changed directly by the noise delta.



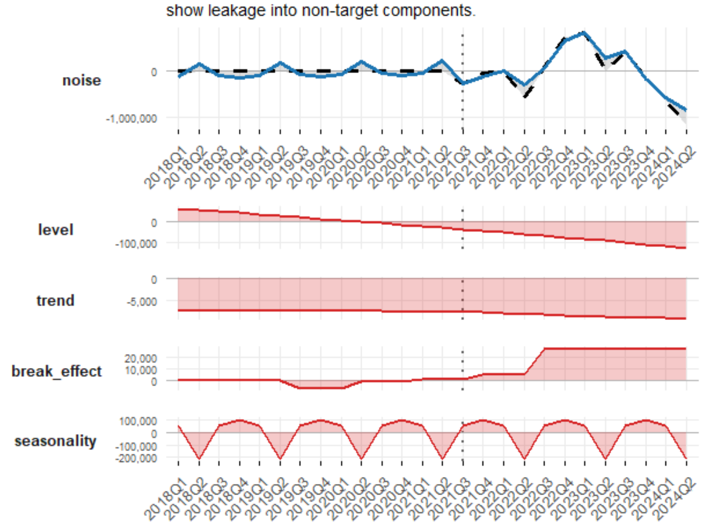
BSTS Decomposition of Perturbed Synthetic CY Series (Stage_4_Noise)

Refitted decomposition after introducing the noise perturbation into the synthetic CY series.



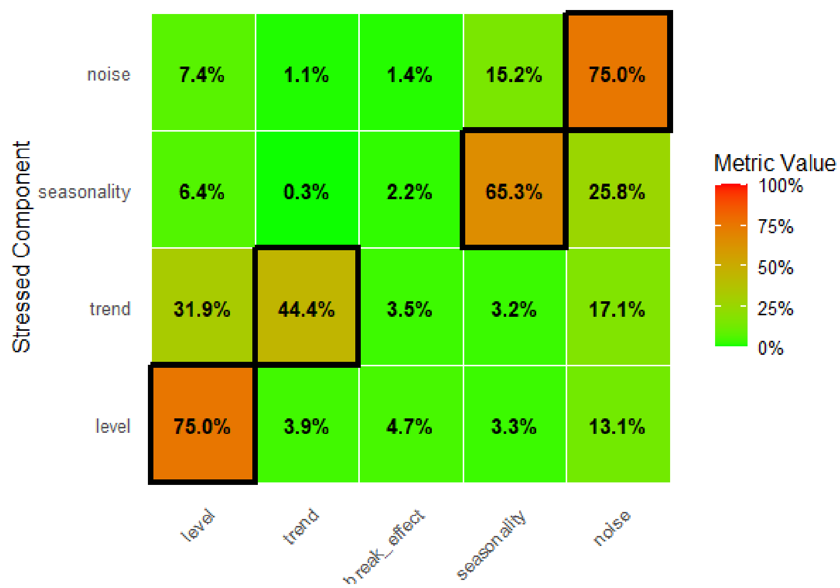
Difference Decomposition: Estimated Attribution versus True Noise Perturbation (CY | Stage_4_Noise)

Noise panel shows truth versus estimated attribution. Other panels show leakage into non-target components.



Recovery Error / Cross-Component Leakage Matrix (CY)

Rows are stressed components. Columns are decomposed components. Diagonal cells are Recovery Error; off-diagonal cells are Cross-Component Leakage.



Experiment 4:

Assumptions:

Data / Modeling Assumptions:
Series Construction:
- Incremental loss triangle derived from cumulative data
- CY series: diagonal aggregation of incremental triangle
- AY series: fixed development age slices from cumulative triangle
- Quarterly frequency assumed ($m = 4$)

Baseline Modeling (CY Level):
- BSTS model fitted to CY series:
  - Local linear trend
  - Seasonal component (quarterly)
  - Step-function break regressors

Structural Break Specification (BSTS models):
- **Candidate breakpoints:**
  - Step functions over time index
  - Restricted to ~25%–75% of sample
  - Spaced every 3 periods
- **Break inclusion:**
  - Determined via spike-and-slab prior
  - Number and magnitude inferred from posterior

CY-Level Perturbations:
- Single-component shocks applied to CY decomposition:
  - Level, trend, seasonal, or noise
- Perturbations introduced at a fixed intervention point
- Perturbed CY series reconstructed from modified components

CY → AY Mapping:
- Perturbed CY increments redistributed into triangle structure
- Allocation proportional across contributing AY cells
- Produces perturbed incremental triangle consistent with CY shocks

---

AY-Level Modeling:
- AY series recomputed from perturbed triangle
- Same BSTS specification applied independently to each AY slice:
  - Local linear trend
  - Seasonal component
  - Step-function break regressors with spike-and-slab selection

---

Attribution:
- Compare baseline vs perturbed AY decompositions
- Quantify propagation of CY shocks into AY components

- Results summarized as percentage allocation across components

---

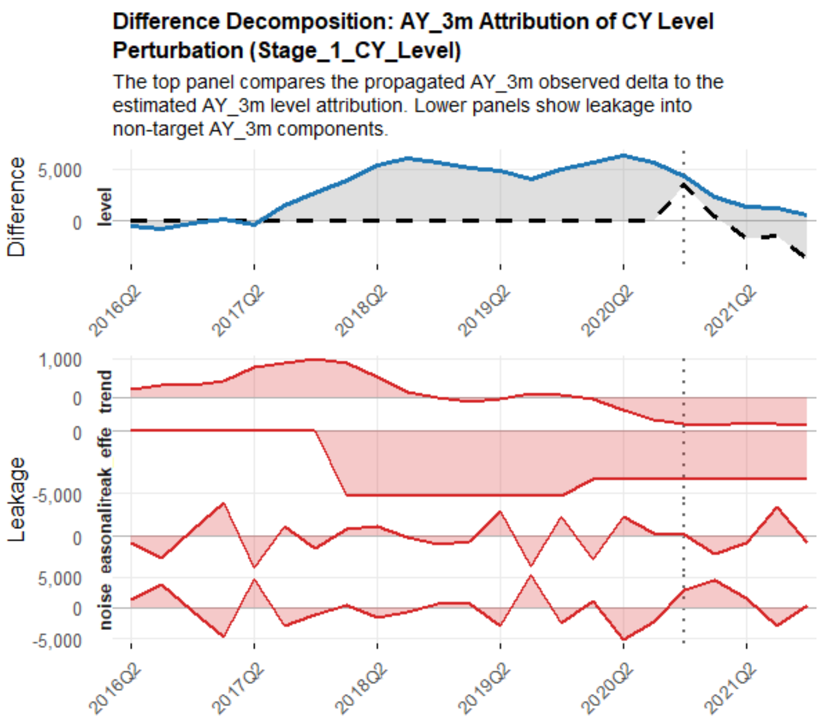
Estimation:

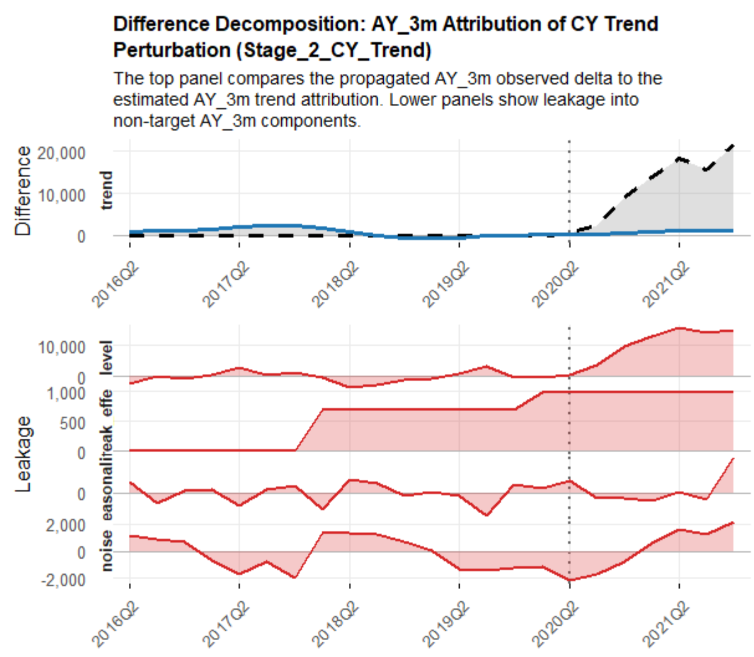
- BSTS estimated via MCMC:
  - 2,500 iterations
  - 20% burn-in

Perturbation Decomposition Results (AY → CY):

(Note: Only difference decomposition plots and component attribution matrices shown for brevity)

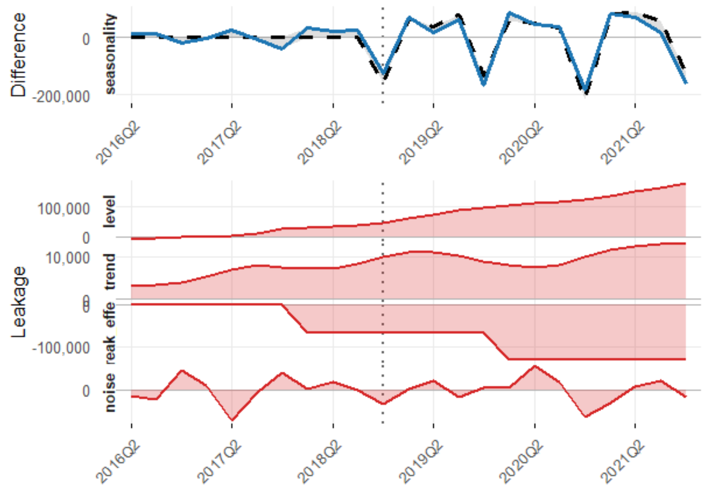
AY (3 months):





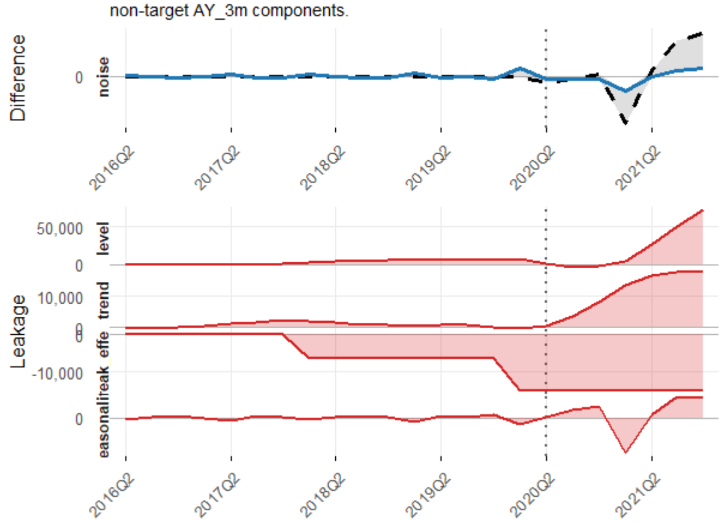
Difference Decomposition: AY_3m Attribution of CY Seasonality Perturbation (Stage_3_CY_Seasonality)

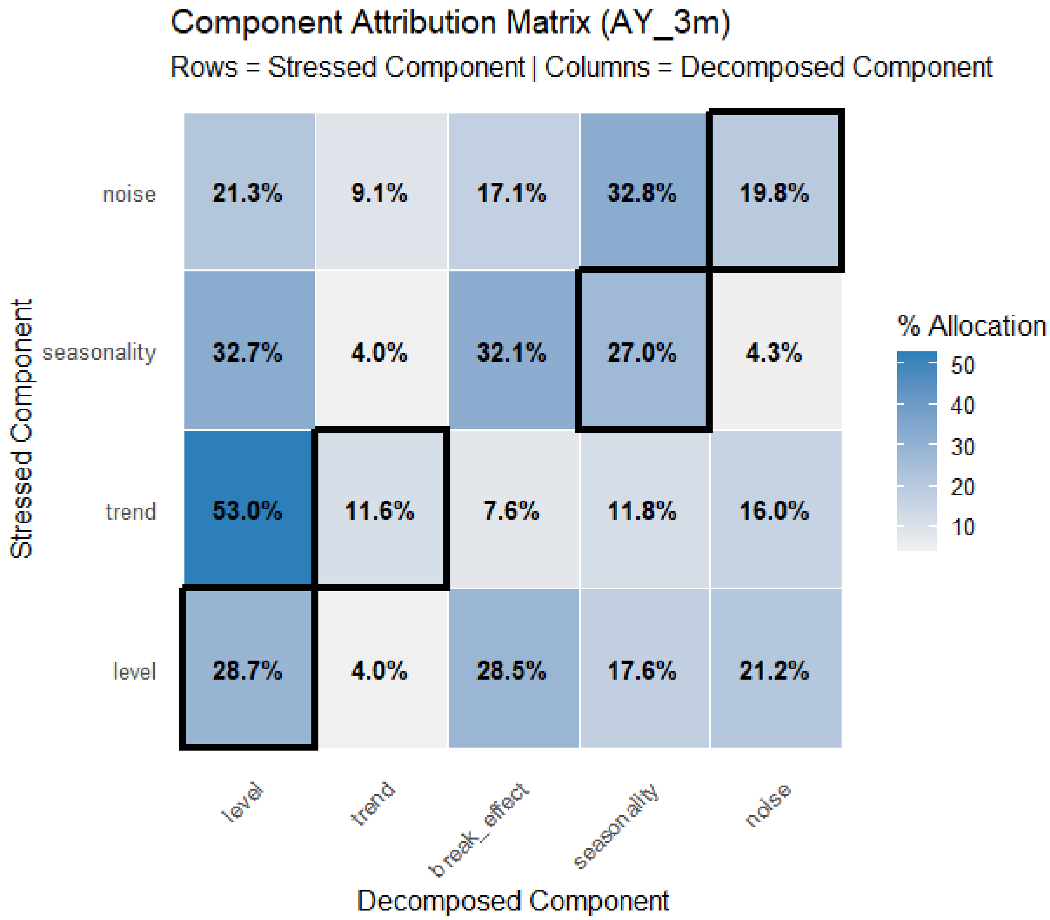
The top panel compares the propagated AY_3m observed delta to the estimated AY_3m seasonality attribution. Lower panels show leakage into non-target AY_3m components.



Difference Decomposition: AY_3m Attribution of CY Noise Perturbation (Stage_4_CY_Noise)

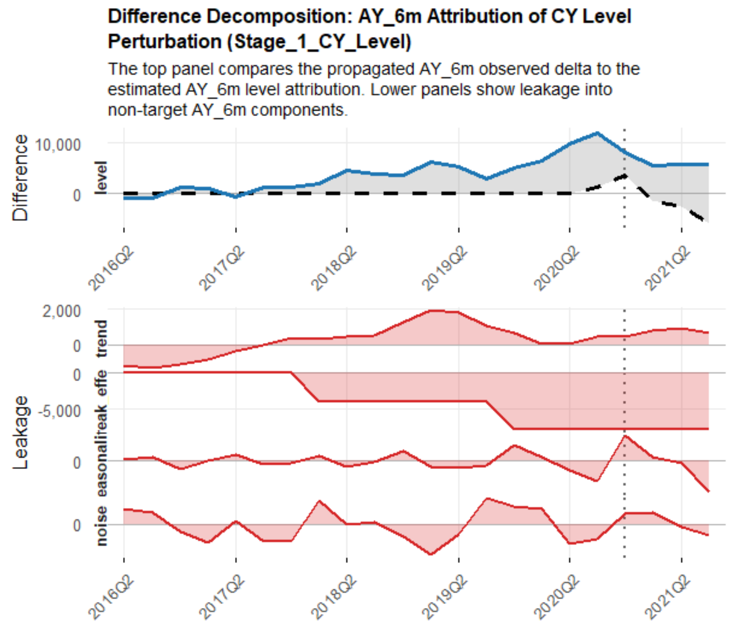
The top panel compares the propagated AY_3m observed delta to the estimated AY_3m noise attribution. Lower panels show leakage into non-target AY_3m components.

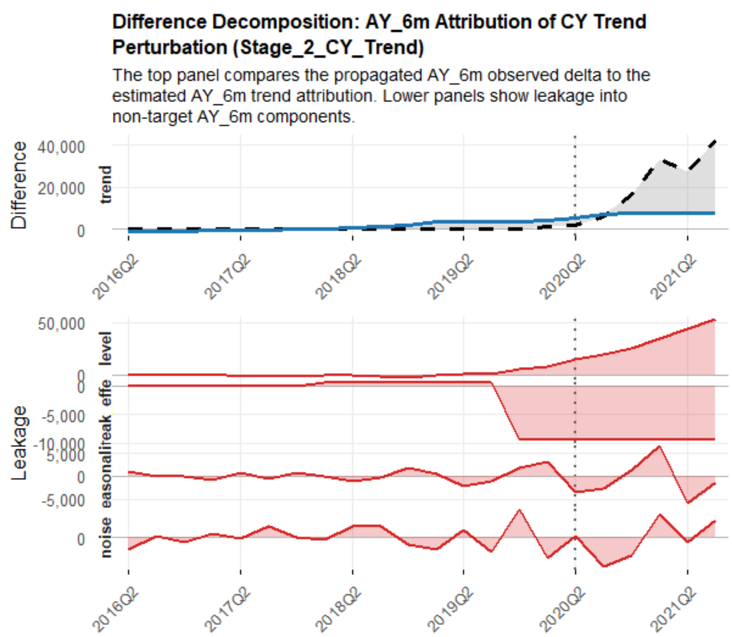




Component Attribution Matrix (AY_3m)
Rows = Stressed Component | Columns = Decomposed Component
Stressed Component
noise
seasonality
trend
level
21.3%
9.1%
17.1%
32.8%
19.8%
32.7%
4.0%
32.1%
27.0%
4.3%
53.0%
11.6%
7.6%
11.8%
16.0%
28.7%
4.0%
28.5%
17.6%
21.2%
level
trend
break_effect
seasonality
noise
Decomposed Component
% Allocation
50
40
30
20
10

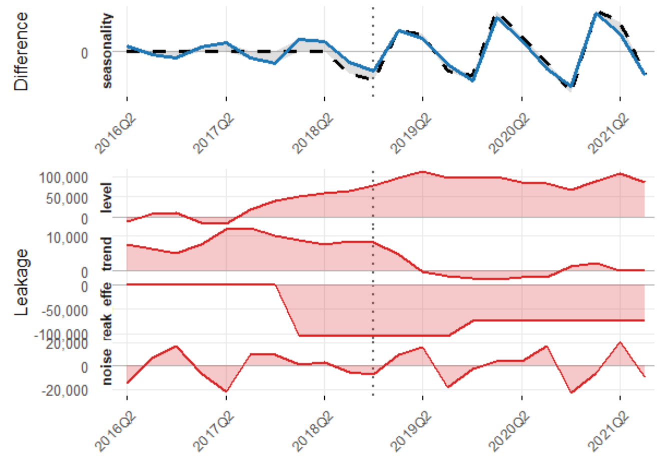
AY (6 months):


Difference Decomposition: AY_6m Attribution of CY Level Perturbation (Stage_1_CY_Level)
The top panel compares the propagated AY_6m observed delta to the estimated AY_6m level attribution. Lower panels show leakage into non-target AY_6m components.
Difference
level
Leakage
trend
noise
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2
2021Q2


Difference Decomposition: AY_6m Attribution of CY Trend Perturbation (Stage_2_CY_Trend)
The top panel compares the propagated AY_6m observed delta to the estimated AY_6m trend attribution. Lower panels show leakage into non-target AY_6m components.
Difference
trend
Leakage
level
noise
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2
2021Q2

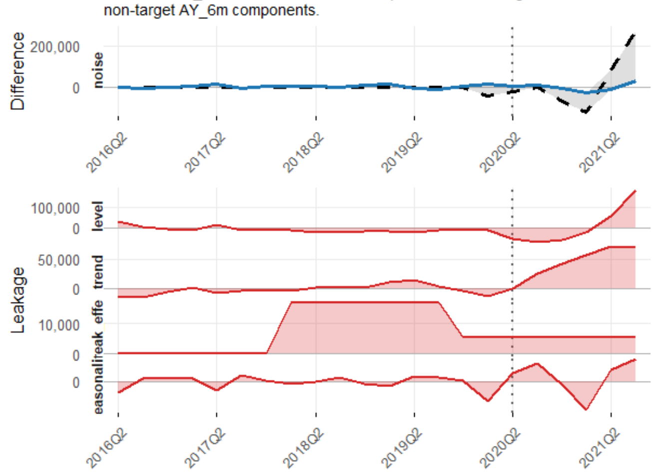
Difference Decomposition: AY_6m Attribution of CY Seasonality Perturbation (Stage_3_CY_Seasonality)

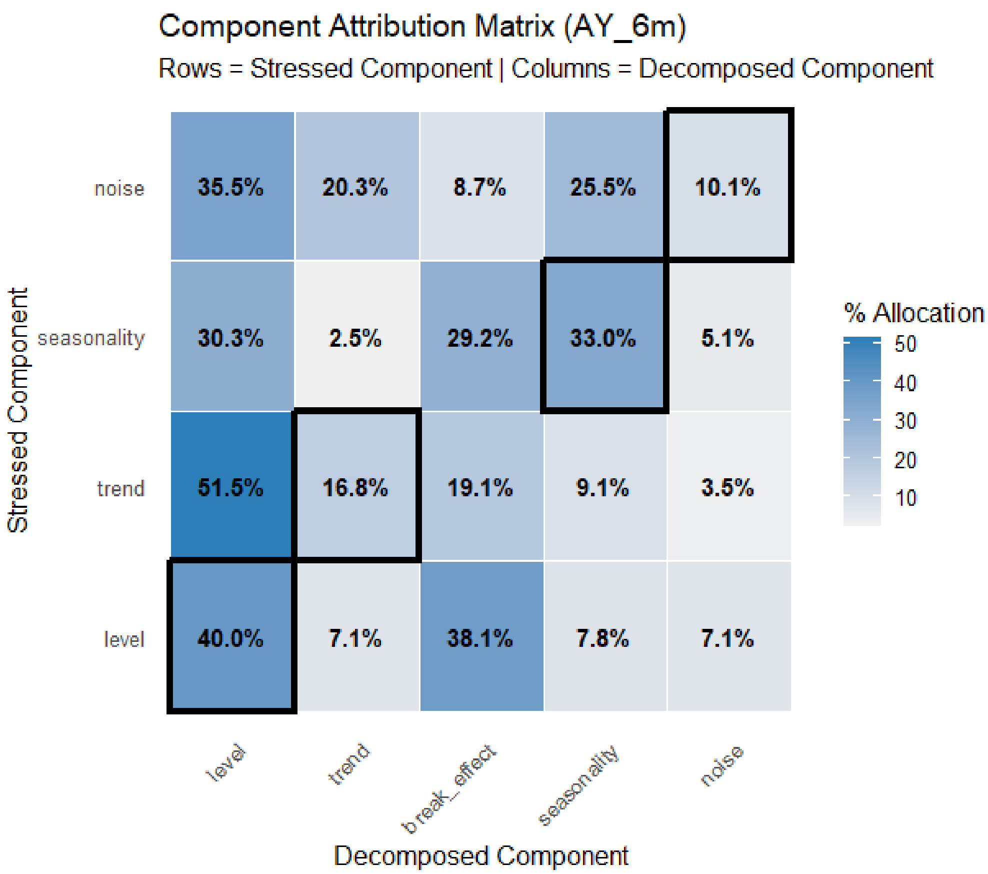
The top panel compares the propagated AY_6m observed delta to the estimated AY_6m seasonality attribution. Lower panels show leakage into non-target AY_6m components.



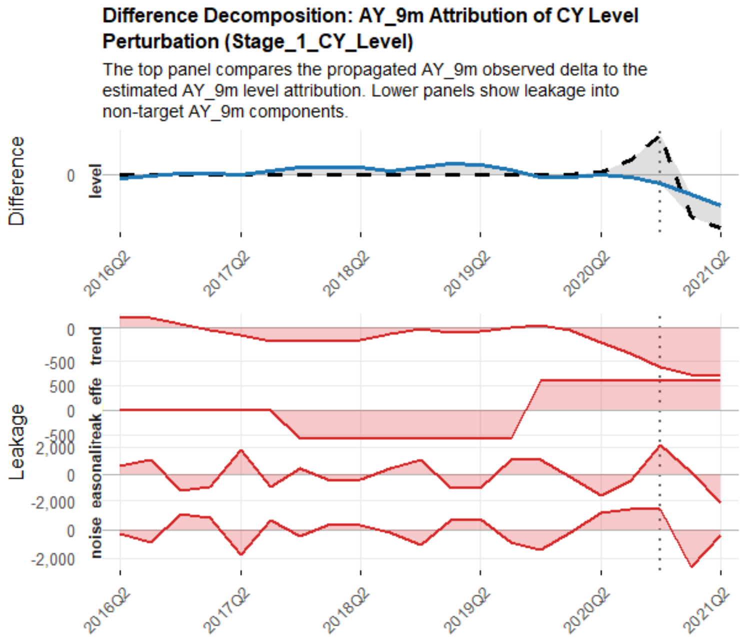
Difference Decomposition: AY_6m Attribution of CY Noise Perturbation (Stage_4_CY_Noise)

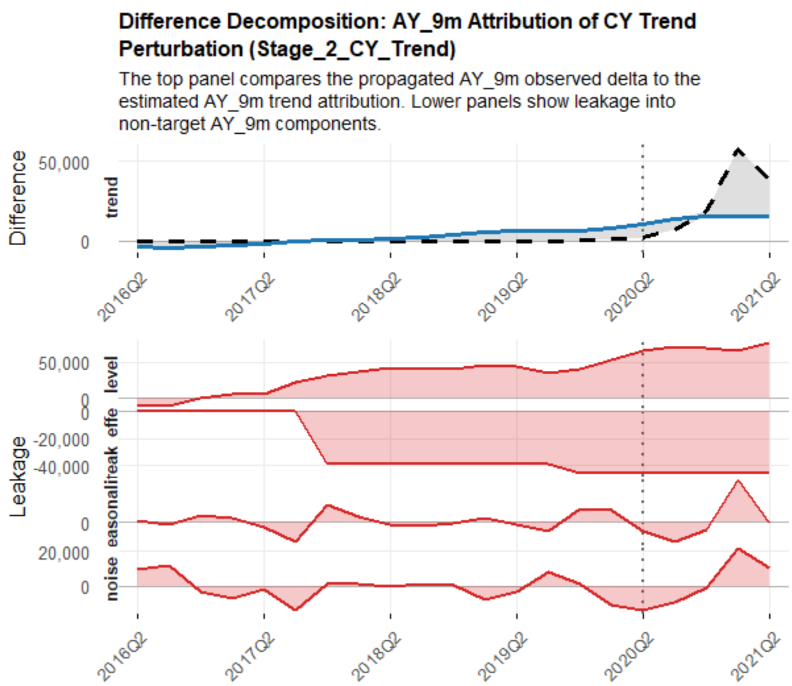
The top panel compares the propagated AY_6m observed delta to the estimated AY_6m noise attribution. Lower panels show leakage into non-target AY_6m components.




Component Attribution Matrix (AY_6m)
Rows = Stressed Component | Columns = Decomposed Component
Stressed Component
noise
seasonality
trend
level
35.5%
20.3%
8.7%
25.5%
10.1%
30.3%
2.5%
29.2%
33.0%
5.1%
51.5%
16.8%
19.1%
9.1%
3.5%
40.0%
7.1%
38.1%
7.8%
7.1%
level
trend
break_effect
seasonality
noise
Decomposed Component
% Allocation
50
40
30
20
10

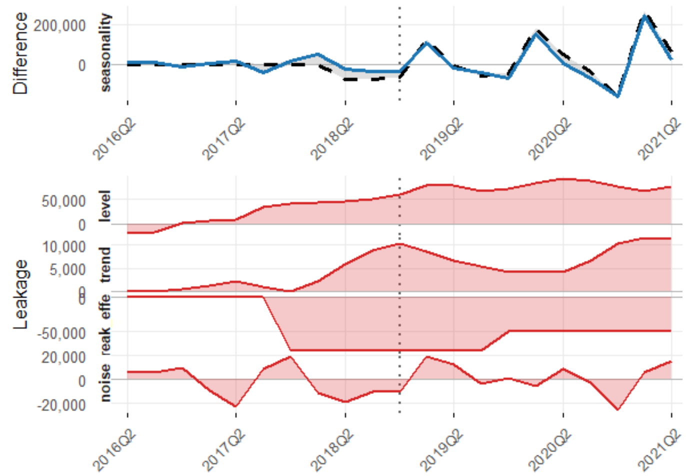
AY (9 months):


Difference Decomposition: AY_9m Attribution of CY Level Perturbation (Stage_1_CY_Level)
The top panel compares the propagated AY_9m observed delta to the estimated AY_9m level attribution. Lower panels show leakage into non-target AY_9m components.
Difference
level
0
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2
2021Q2
Leakage
trend
effe
easonalireak
noise
0
-500
500
0
-500
2,000
0
-2,000
0
-2,000


Difference Decomposition: AY_9m Attribution of CY Trend Perturbation (Stage_2_CY_Trend)
The top panel compares the propagated AY_9m observed delta to the estimated AY_9m trend attribution. Lower panels show leakage into non-target AY_9m components.
Difference
trend
50,000
0
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2
2021Q2
Leakage
level
effe
easonalireak
noise
50,000
0
0
-20,000
-40,000
0
20,000
0

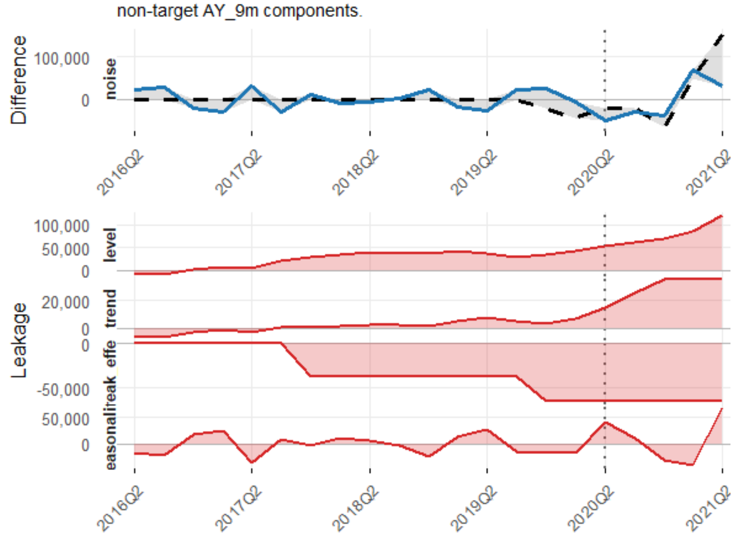
Difference Decomposition: AY_9m Attribution of CY Seasonality Perturbation (Stage_3_CY_Seasonality)

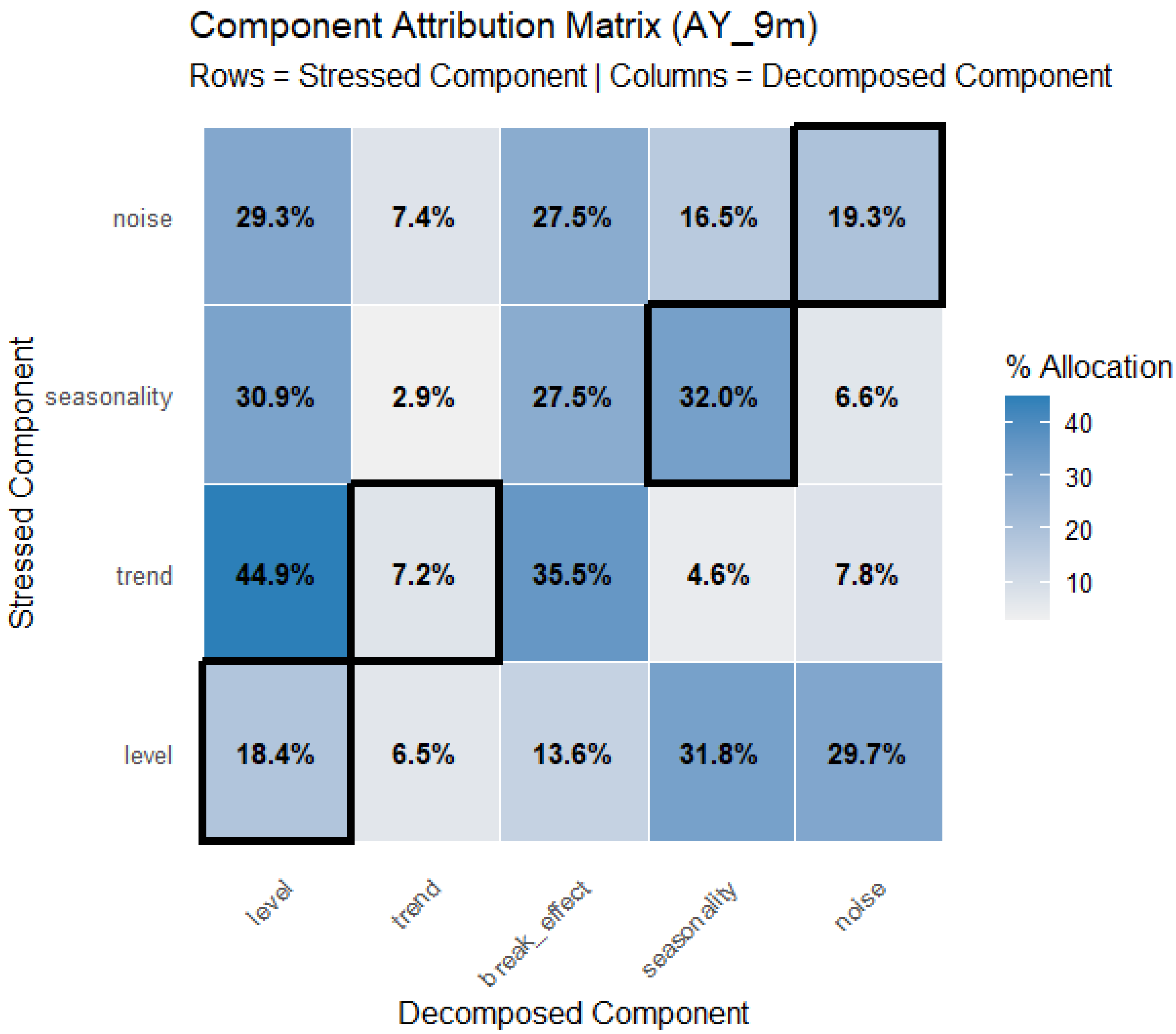
The top panel compares the propagated AY_9m observed delta to the estimated AY_9m seasonality attribution. Lower panels show leakage into non-target AY_9m components.



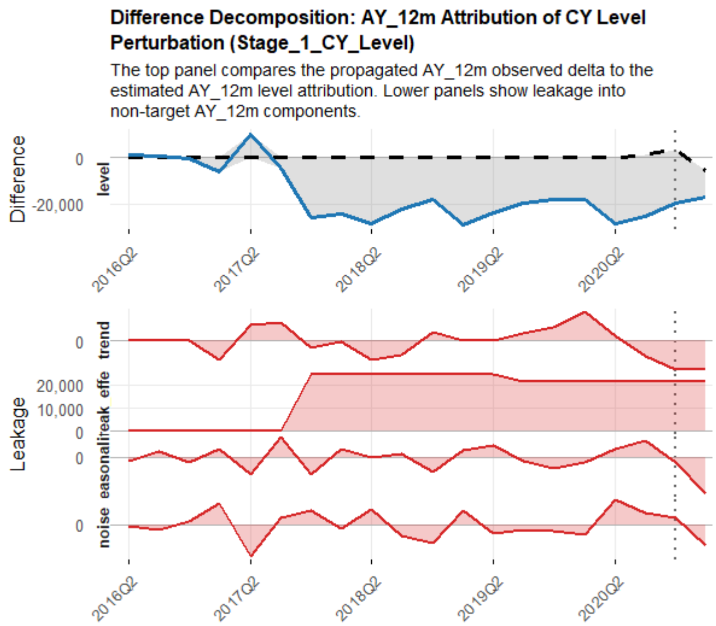
Difference Decomposition: AY_9m Attribution of CY Noise Perturbation (Stage_4_CY_Noise)

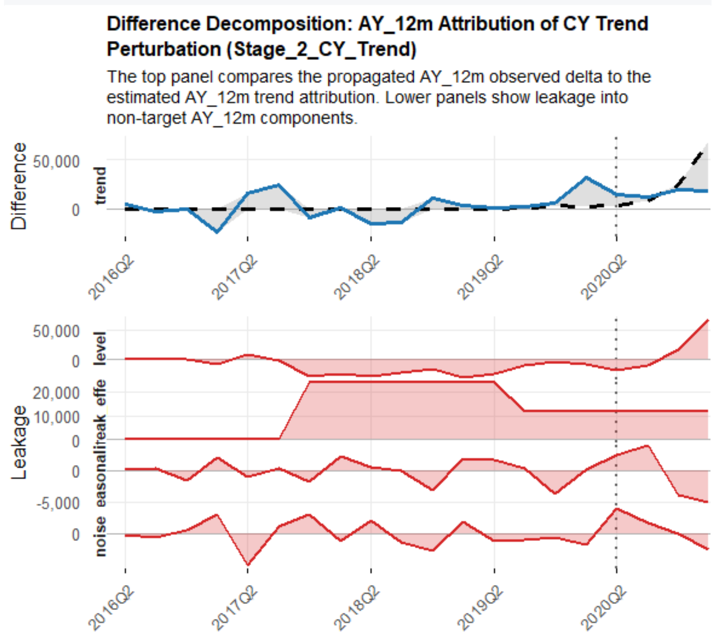
The top panel compares the propagated AY_9m observed delta to the estimated AY_9m noise attribution. Lower panels show leakage into non-target AY_9m components.




Component Attribution Matrix (AY_9m)
Rows = Stressed Component | Columns = Decomposed Component
noise
seasonality
trend
level
29.3%
7.4%
27.5%
16.5%
19.3%
30.9%
2.9%
27.5%
32.0%
6.6%
44.9%
7.2%
35.5%
4.6%
7.8%
18.4%
6.5%
13.6%
31.8%
29.7%
level
trend
break_effect
seasonality
noise
Decomposed Component
Stressed Component
% Allocation
40
30
20
10

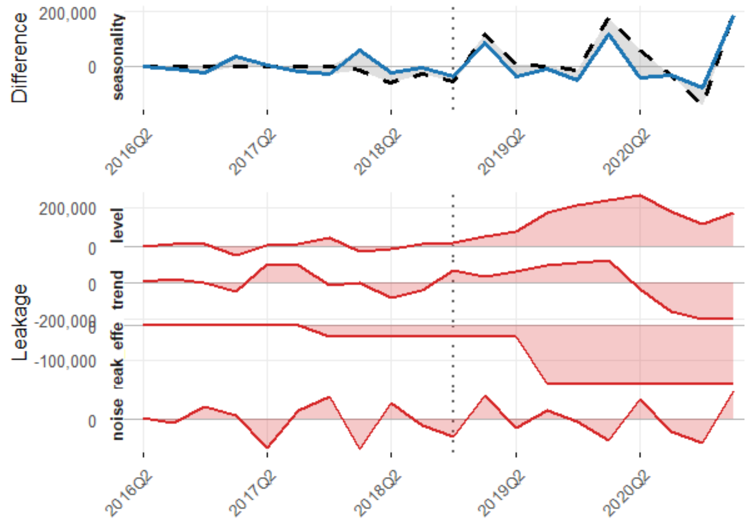
AY (12 months):


Difference Decomposition: AY_12m Attribution of CY Level Perturbation (Stage_1_CY_Level)
The top panel compares the propagated AY_12m observed delta to the estimated AY_12m level attribution. Lower panels show leakage into non-target AY_12m components.
Difference
level
0
-20,000
Leakage
trend
noise
20,000
10,000
0
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2


Difference Decomposition: AY_12m Attribution of CY Trend Perturbation (Stage_2_CY_Trend)
The top panel compares the propagated AY_12m observed delta to the estimated AY_12m trend attribution. Lower panels show leakage into non-target AY_12m components.
Difference
trend
50,000
0
Leakage
level
noise
20,000
10,000
-5,000
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2

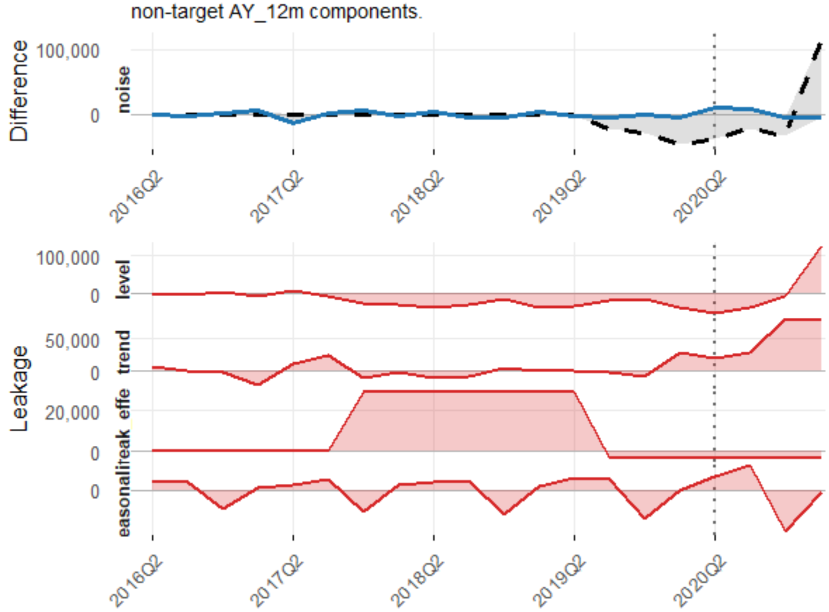
Difference Decomposition: AY_12m Attribution of CY Seasonality Perturbation (Stage_3_CY_Seasonality)

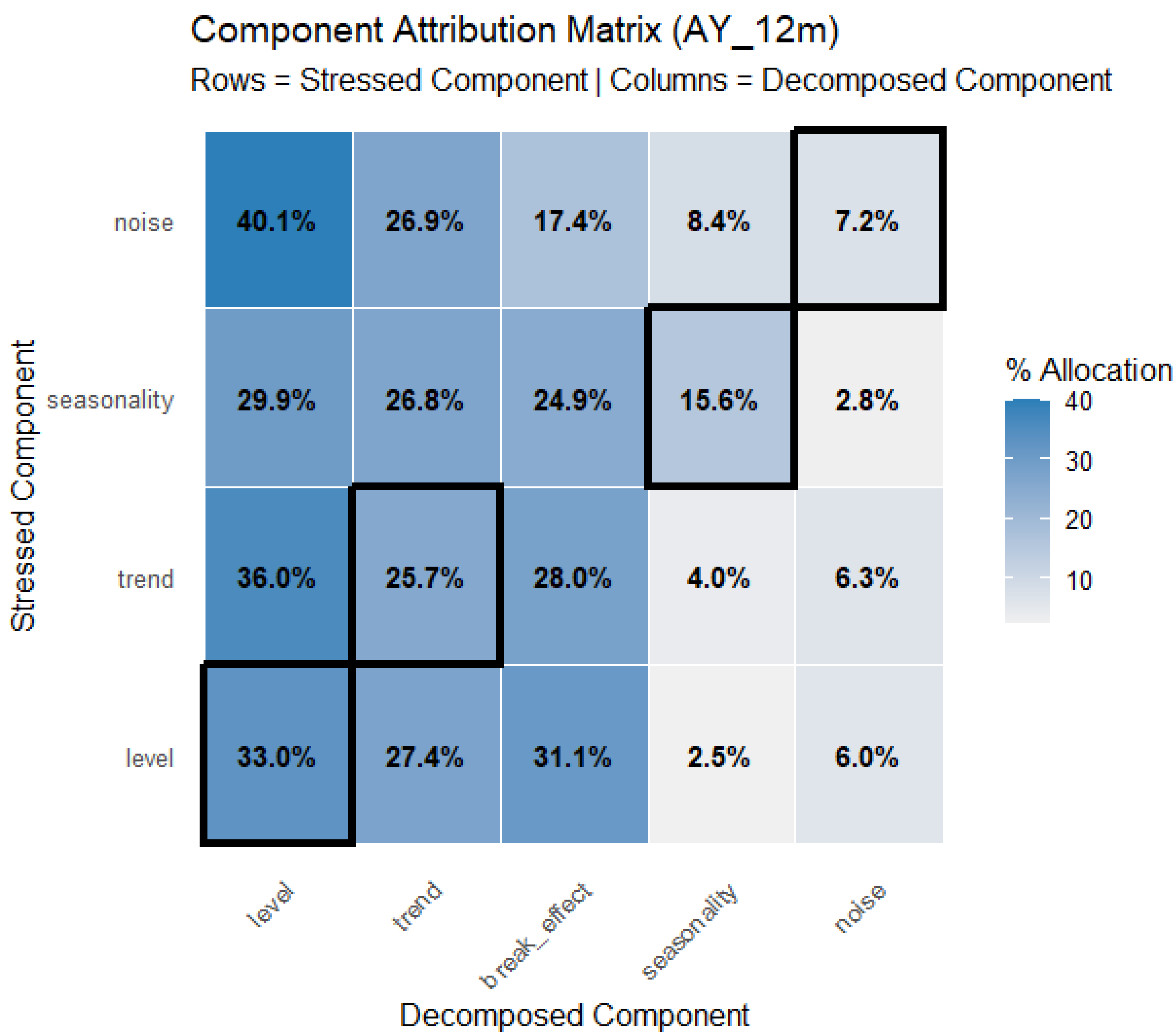
The top panel compares the propagated AY_12m observed delta to the estimated AY_12m seasonality attribution. Lower panels show leakage into non-target AY_12m components.



Difference Decomposition: AY_12m Attribution of CY Noise Perturbation (Stage_4_CY_Noise)

The top panel compares the propagated AY_12m observed delta to the estimated AY_12m noise attribution. Lower panels show leakage into non-target AY_12m components.

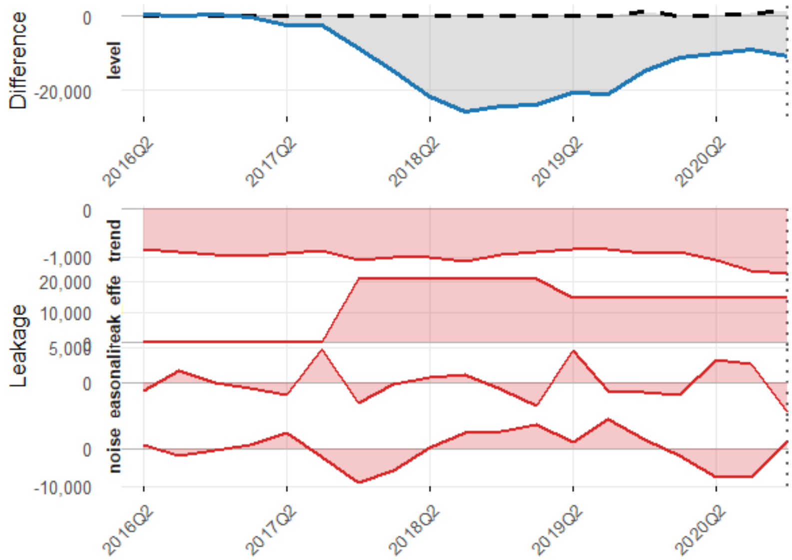



Component Attribution Matrix (AY_12m)
Rows = Stressed Component | Columns = Decomposed Component
noise
seasonality
trend
level
Stressed Component
40.1%
26.9%
17.4%
8.4%
7.2%
29.9%
26.8%
24.9%
15.6%
2.8%
36.0%
25.7%
28.0%
4.0%
6.3%
33.0%
27.4%
31.1%
2.5%
6.0%
level
trend
break_effect
seasonality
noise
Decomposed Component
% Allocation
40
30
20
10

AY (15 months):

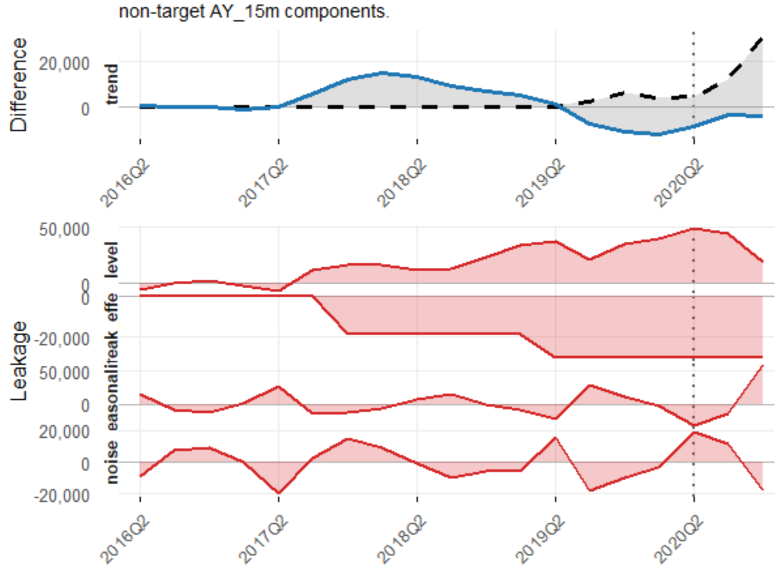
Difference Decomposition: AY_15m Attribution of CY Level Perturbation (Stage_1_CY_Level)

The top panel compares the propagated AY_15m observed delta to the estimated AY_15m level attribution. Lower panels show leakage into non-target AY_15m components.



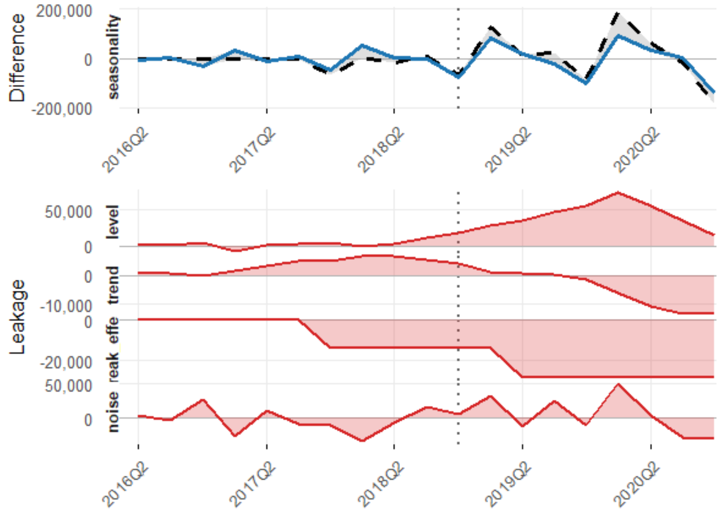
Difference Decomposition: AY_15m Attribution of CY Trend Perturbation (Stage_2_CY_Trend)

The top panel compares the propagated AY_15m observed delta to the estimated AY_15m trend attribution. Lower panels show leakage into non-target AY_15m components.



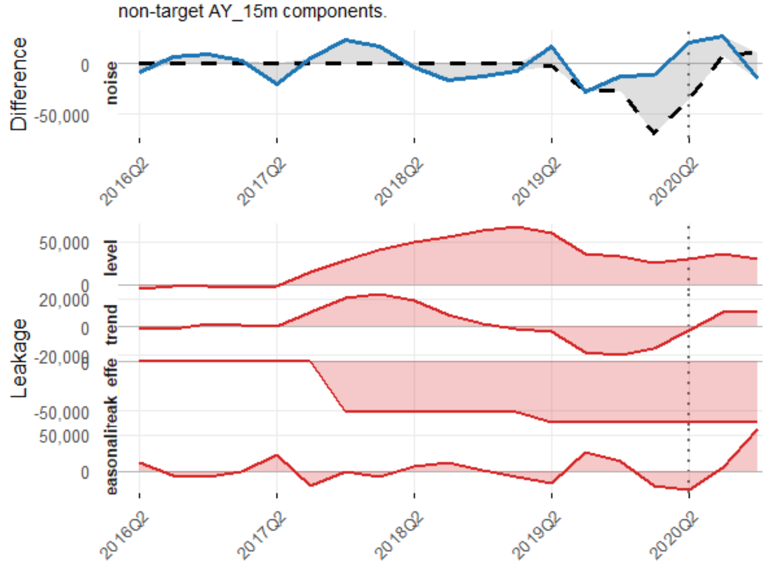
Difference Decomposition: AY_15m Attribution of CY Seasonality Perturbation (Stage_3_CY_Seasonality)

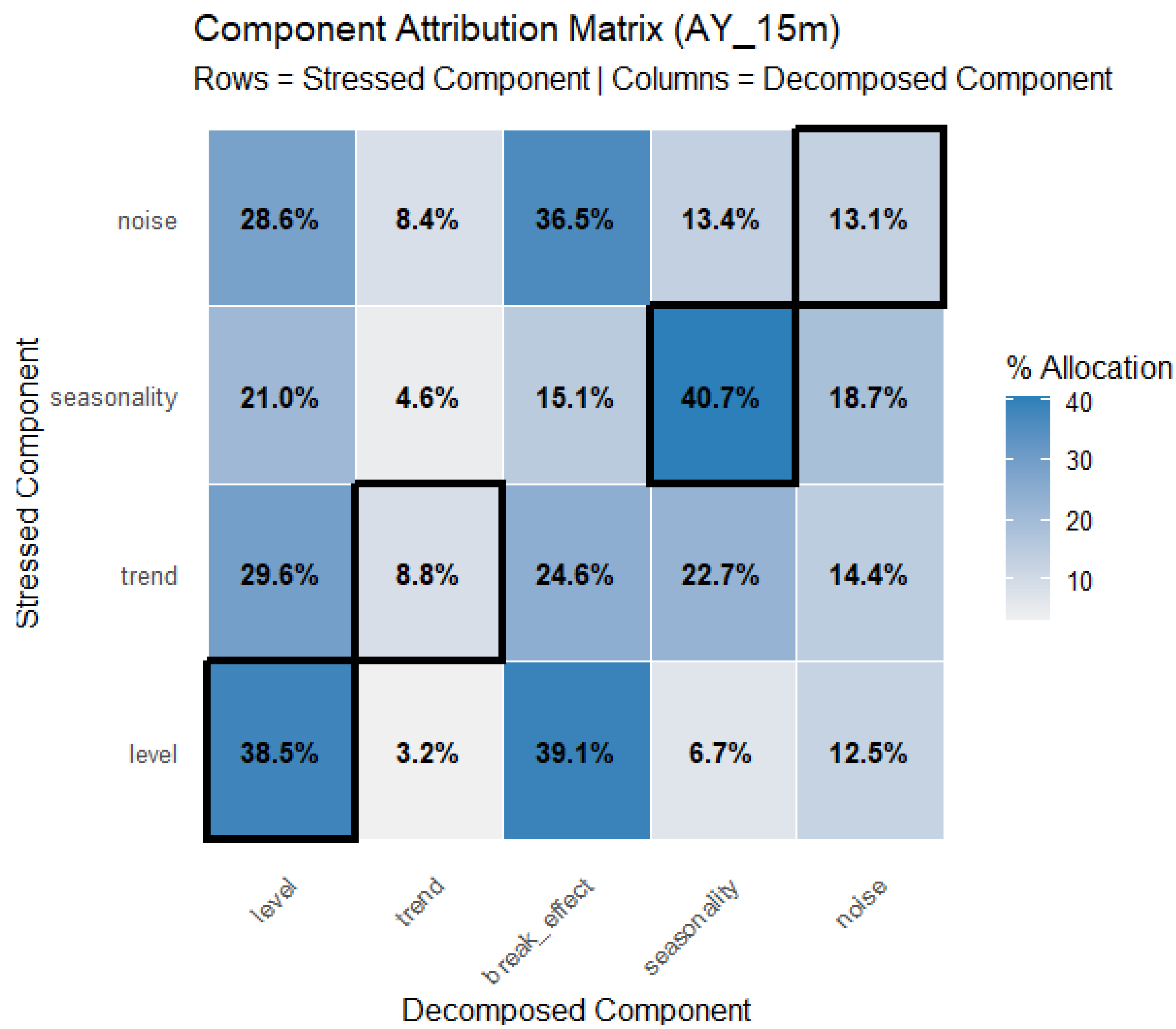
The top panel compares the propagated AY_15m observed delta to the estimated AY_15m seasonality attribution. Lower panels show leakage into non-target AY_15m components.



Difference Decomposition: AY_15m Attribution of CY Noise Perturbation (Stage_4_CY_Noise)

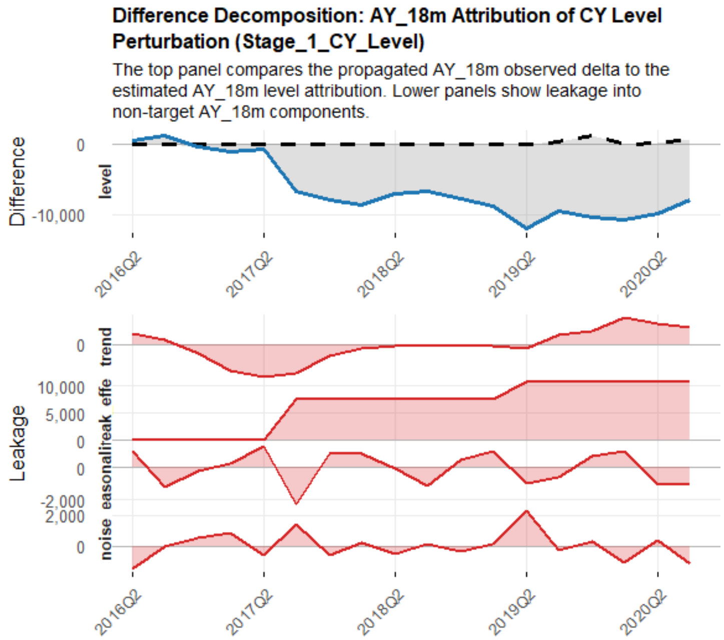
The top panel compares the propagated AY_15m observed delta to the estimated AY_15m noise attribution. Lower panels show leakage into non-target AY_15m components.

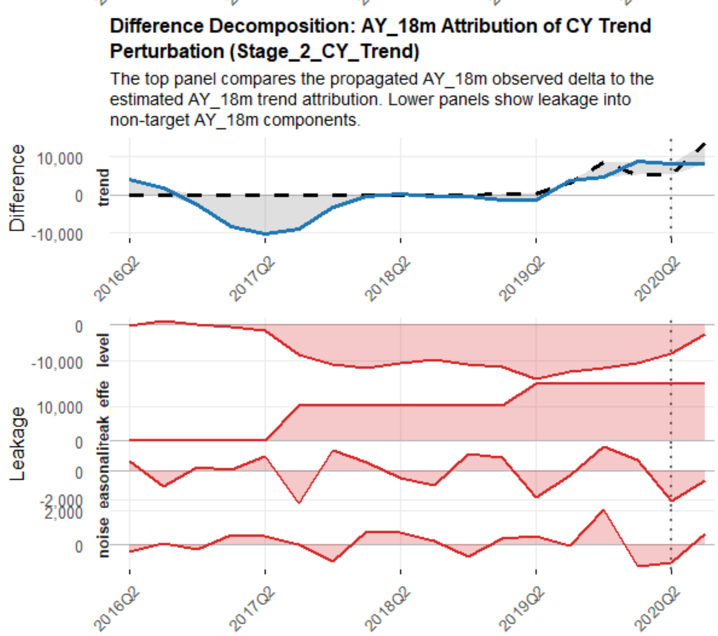




Component Attribution Matrix (AY_15m)
Rows = Stressed Component | Columns = Decomposed Component
Stressed Component
noise
seasonality
trend
level
28.6%
8.4%
36.5%
13.4%
13.1%
21.0%
4.6%
15.1%
40.7%
18.7%
29.6%
8.8%
24.6%
22.7%
14.4%
38.5%
3.2%
39.1%
6.7%
12.5%
level
trend
break_effect
seasonality
noise
Decomposed Component
% Allocation
40
30
20
10

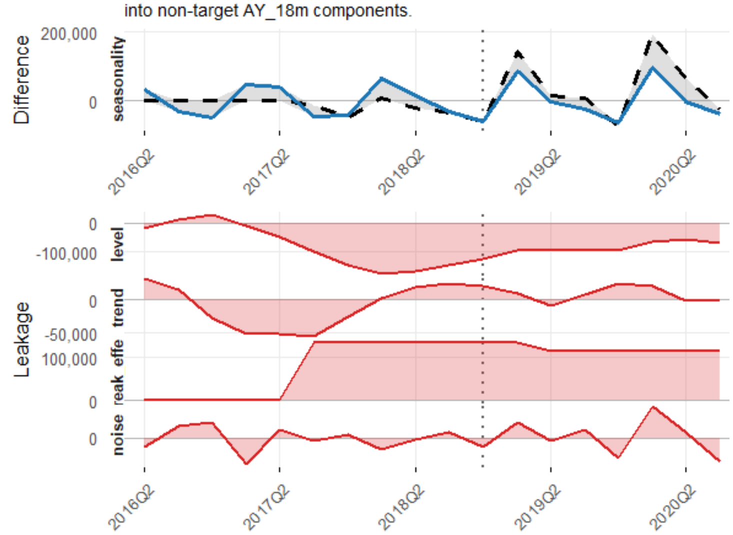
AY (18 months):


Difference Decomposition: AY_18m Attribution of CY Level Perturbation (Stage_1_CY_Level)
The top panel compares the propagated AY_18m observed delta to the estimated AY_18m level attribution. Lower panels show leakage into non-target AY_18m components.
Difference
level
Leakage
trend
noise
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2


Difference Decomposition: AY_18m Attribution of CY Trend Perturbation (Stage_2_CY_Trend)
The top panel compares the propagated AY_18m observed delta to the estimated AY_18m trend attribution. Lower panels show leakage into non-target AY_18m components.
Difference
trend
Leakage
level
noise
2016Q2
2017Q2
2018Q2
2019Q2
2020Q2

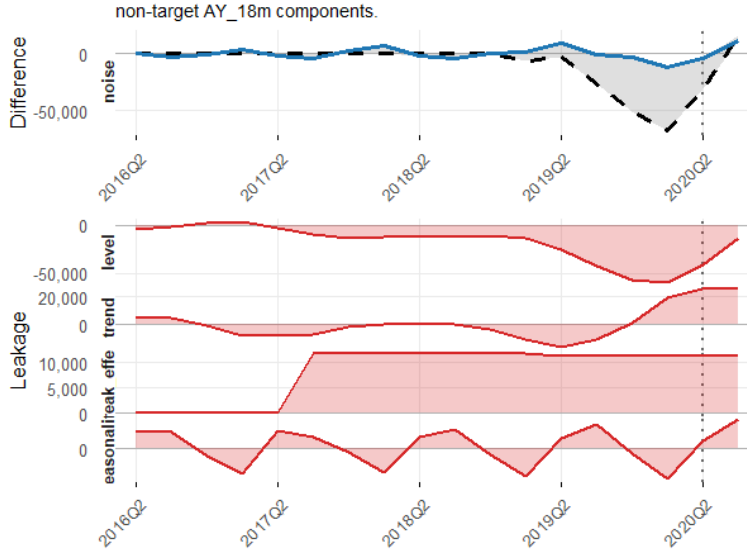
Difference Decomposition: AY_18m Attribution of CY Seasonality Perturbation (Stage_3_CY_Seasonality)

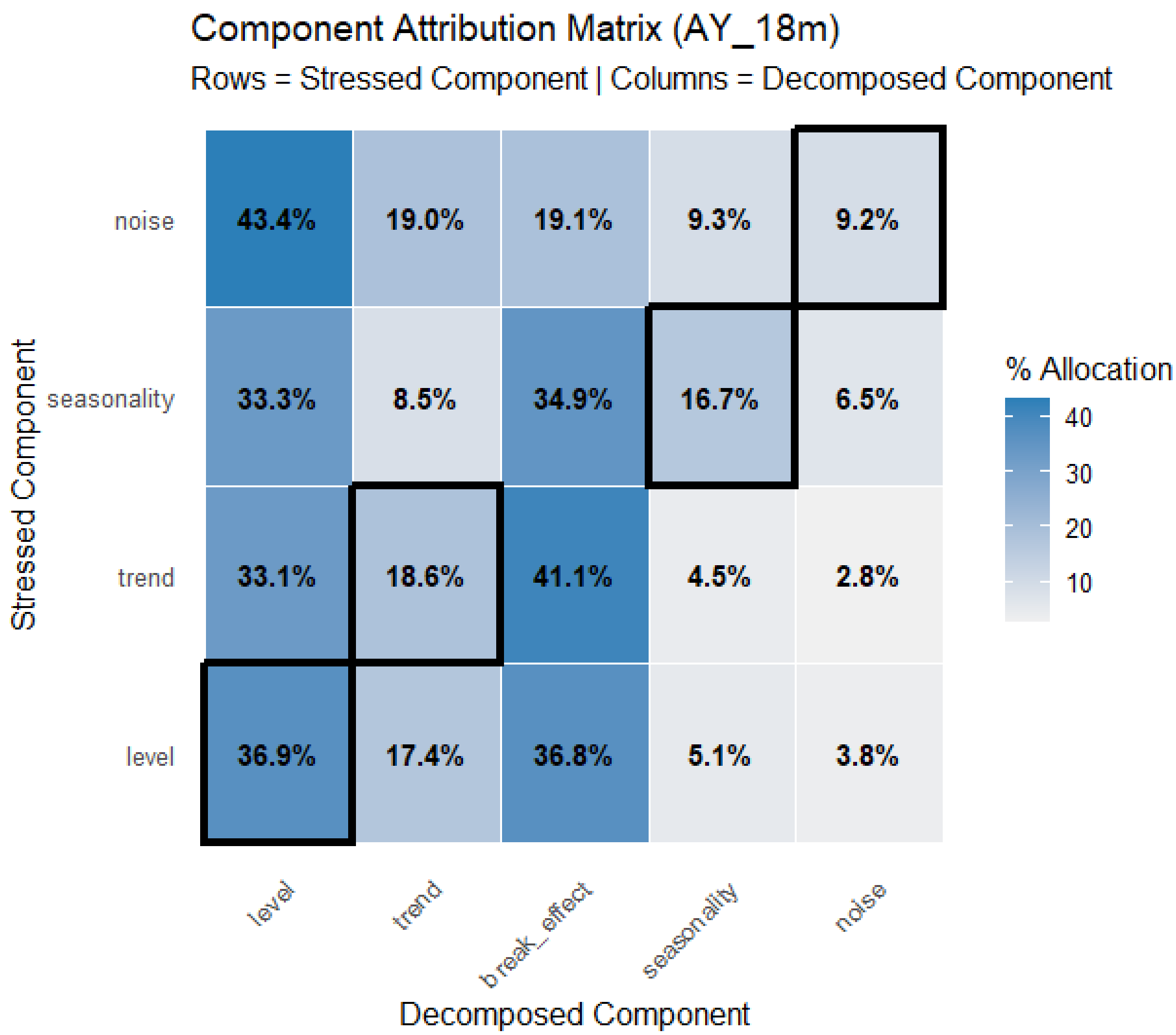
The top panel compares the propagated AY_18m observed delta to the estimated AY_18m seasonality attribution. Lower panels show leakage into non-target AY_18m components.



Difference Decomposition: AY_18m Attribution of CY Noise Perturbation (Stage_4_CY_Noise)

The top panel compares the propagated AY_18m observed delta to the estimated AY_18m noise attribution. Lower panels show leakage into non-target AY_18m components.




Component Attribution Matrix (AY_18m)
Rows = Stressed Component | Columns = Decomposed Component
Stressed Component
noise
seasonality
trend
level
43.4%
19.0%
19.1%
9.3%
9.2%
33.3%
8.5%
34.9%
16.7%
6.5%
33.1%
18.6%
41.1%
4.5%
2.8%
36.9%
17.4%
36.8%
5.1%
3.8%
level
trend
break_effect
seasonality
noise
Decomposed Component
% Allocation
40
30
20
10